Key CONCEPTS

To determine the distance from a given point to a line whose equation is given,

a) write an equation for the perpendicular from the given point to the given line

b) find the coordinates of the point of intersection of the perpendicular and the given line

c) use the distance formula

Communicate Your Understanding

1. Describe how you would find the distance from the point P(4, 2) to the line $x - y = 0$.

2. The equation of a horizontal line is $y = 5$. Explain how to find the distance from the point P(3, 1) to the line $y = 5$.

Practice

A

1. Find the exact value of the shortest distance from the origin to each line.

a) $y = x - 4$ **b)** $y = x + 6$ **c)** $y = 3x - 10$ **d)** $y = -2x - 5$

e) $y = -2$ **f)** $x - 3 = 0$ **g)** $y = \frac{2}{3}x - 13$ **h)** $y = -\frac{1}{2}x - 5$

2. Find the shortest distance from the origin to each line, to the nearest tenth.

a) $x - y = 3$ **b)** $x + y + 2 = 0$ **c)** $x + 2y = 4$ **d)** $3x - 4y = -6$

e) $6x + 8y - 5 = 0$ **f)** $4x = 6y + 13$ **g)** $2x + 3y = 9$ **h)** $x - 5y + 11 = 0$

3. Find the shortest distance from the given point to the given line. Round to the nearest tenth, if necessary.

a) (2, 2) and $y = x + 1$ **b)** (5, 0) and $y = 0.5x + 5$

c) (3, 1) and $x + y = -2$ **d)** (–3, 0) and $x - y = -10$

e) (3, –1) and $2x - y + 3 = 0$ **f)** (–1, 2) and $x - 4y + 1 = 0$

g) (0, –1) and $5x + 2y + 3 = 0$ **h)** (–2, –1) and $2x + y + 3 = 0$

4. Find the distance from A(–2, –2) to the line joining B(5, 2) and C(–1, 4), to the nearest hundredth.

5. **Communication** Calculate the distance from the point K(–10, –13) to the line $8x - 5y + 15 = 0$. Explain the answer in terms of the relationship between the point and the line.

6. Find the exact distance from the point D(4, –2) to the line segment joining the points E(1, 3) and F(–4, –2).

7. Find the distance from the point P(4, 1) to the line segment joining the points Q(1, −8) and R(−4, 2) as
a) an exact value
b) an approximate value, to the nearest tenth

Applications and Problem Solving

8. Find the distance from the point P(3, 5) to the line $x = -2$.

9. Find the distance from the point Q(−2, −2) to the line $y = 3$.

B

10. A line has y-intercept of 8 and x-intercept of −12. What is the shortest distance from the origin to this line, to the nearest tenth?

11. Measurement A triangle has vertices A(−4, −2), B(2, 4), and C(3, −3).
a) Classify the triangle by side length.
b) Verify that exactly one vertex of the triangle lies on the perpendicular bisector of the opposite side.
c) Determine the area of the triangle.

12. Measurement **a)** For the triangle whose vertices are A(1, −1), B(0, 5), and C(−3, 0), determine the length of each altitude.
b) Calculate the area of △ABC.

13. The equation of a circle with centre O(0, 0) is $x^2 + y^2 = 10$. The points P(1, 3) and Q(−3, −1) are endpoints of chord PQ. Find the exact distance from the centre of the circle to the chord.

14. The equation of a circle with centre O(0, 0) is $x^2 + y^2 = 17$. The points M(1, 4) and N(4, −1) are endpoints of chord MN. Find the distance from the centre of the circle to the chord, to the nearest tenth.

C

15. Measurement **a)** Find the distance from the point of intersection of the lines $2x + 3y = 10$ and $3x - y = 4$ to the line $5x - 6y = 1$, to the nearest hundredth.
b) Find the area of the triangle formed by the three lines, to the nearest hundredth.

16. Measurement Find the area of the trapezoid with vertices K(−4, 3), L(−1, 4), M(10, 1), and N(−5, −4).

17. Write equations for two lines that meet each of the following sets of conditions. Compare your equations with your classmates'.
a) slope is $\frac{1}{2}$; shortest distance from origin is $\sqrt{13}$
b) slope is −3; shortest distance from origin is $\sqrt{5}$

and so they are perpendicular. The triangle is right isosceles. **19.** The equations of the diagonals are $x - y - 2 = 0$ and $2x + 3y - 9 = 0$. These lines intersect at (3, 1). **20. a)** UV has slope 2; WV has slope $-\frac{1}{2}$; UV and WV are perpendicular. **b)** The median has length 5; the hypotenuse UW has length 10; the median is half the length of the hypotenuse. **21. a)** Opposite sides are parallel (two sides have slope $\frac{3}{2}$ and two sides have slope $-\frac{2}{3}$). Adjacent sides are perpendicular and all sides have length $\sqrt{13}$. KLMN is a square. **b)** The midpoints of the diagonals coincide at $(-\frac{1}{2}, \frac{3}{2})$, so the diagonals bisect each other. One diagonal has slope $\frac{1}{5}$ and the other has slope –5. The diagonals are perpendicular. **c)** The diagonals both have length $\sqrt{26}$. **22. a)** isosceles **b)** AC has slope $-\frac{1}{11}$.and the median from B to AC has slope 11; thus, they are perpendicular **23.** No, the perpendicular bisector of UV has equation $x + y - 2 = 0$, which does not contain the point T(2, –1). **24.** One diagonal is vertical, and two segments joining midpoints of adjacent sides are also vertical; the other diagonal has slope $\frac{1}{10}$, and two segments joining midpoints of adjacent sides also have slope $\frac{1}{10}$. **25. a)** kite **b)** Opposite sides are parallel (two sides have slope $-\frac{1}{3}$ and two have slope 3) and adjacent sides are perpendicular. Thus, it is a rectangle. **26. a)** (0, 0), (240, 0), (240, 120), (0, 120), (120, 60) **b)** One direction may be described by a line segment with slope $\frac{1}{2}$ and the other by a line segment with slope $-\frac{1}{2}$. These are not negative reciprocals, so the directions are not perpendicular. **27. a)** $m\text{PS} = m\text{QR} = \frac{1}{2}$, $m\text{PQ} = -3$, $m\text{SR} = -\frac{1}{5}$ **b)** midpoint of PQ is A (–2, –2), midpoint of SR is B(6, 2); $m\text{AB} = \frac{1}{2}$ **c)** $\text{PS} = 2\sqrt{5}$, $\text{QR} = 6\sqrt{5}$, $\text{AB} = 4\sqrt{5}$; PS + QR = 2AB **28.** All sides have length 2. **29. a)** The equations of the medians are $x = 6$, $y = 0$, and $2x + 3y - 12 = 0$; these lines intersect at (6, 0). **b)** The equations of the altitudes are $3x - y - 14 = 0$, $3x - 4y = 0$, and $3x + 2y - 28 = 0$. These lines intersect at $\left(\frac{56}{9}, \frac{14}{3}\right)$. **c)** The equations of the perpendicular bisectors are $3x - y - 20 = 0$, $3x - 4y - 27 = 0$, and $3x + 2y - 13 = 0$. These lines intersect at $\left(\frac{53}{9}, -\frac{7}{3}\right)$. **d)** The slope of the line segment connecting the centroid and the orthocentre is 21. The slope of the line segment connecting the orthocentre and the circumcentre is 21. The three points are collinear.

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1. Two sides of the triangle have slopes –1 and 1, respectively. Thus they are perpendicular. **2.** Opposite sides are parallel and adjacent sides are perpendicular (two sides have slope 1 and two have slope –1). **3.** Opposite sides are parallel (two have slope 0 and two have slope 1). **4.** For example, the line joining the midpoints (15, 35) and (25, 35) has length 10 and slope 0. The corresponding side of the triangle has length 20 and slope 0.

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1. a) (0, 0), (5, 0), (5, 3), (0, 3) **b)** $\sqrt{34}$ **c)** No, since $\sqrt{34}$ is an irrational number. A builder would probably round to the nearest millimetre.

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Practice **1. a)** $\sqrt{8}$ **b)** $\sqrt{18}$ **c)** $\sqrt{10}$ **d)** $\sqrt{5}$ **e)** 2 **f)** 3 **g)** $\sqrt{117}$ **h)** $\sqrt{20}$ **2. a)** 2.1 **b)** 1.4 **c)** 1.8 **d)** 1.2 **e)** 0.5 **f)** 1.8 **g)** 2.5 **h)** 2.2 **3. a)** 0.7 **b)** 6.7 **c)** 4.2 **d)** 4.9 **e)** 4.5 **f)** 1.9 **g)** 0.2 **h)** 0.9 **4.** 6.01 **5.** 0; the point is on the line. **6.** $\sqrt{32}$ **7. a)** $\sqrt{45}$ **b)** 6.7

Applications and Problem Solving **8.** 5 **9.** 5 **10.** 6.7 **11. a)** isosceles **b)** C lies on the perpendicular bisector of AB. **c)** 24 **12. a)** 5.58, 3.78, 3.94 **b)** 11.5 **13.** $\sqrt{2}$ **14.** 2.9 **15. a)** 0.38 **b)** 0.14 **16.** 60 **17. a)** $y = \frac{1}{2}x + \frac{\sqrt{65}}{2}$, $y = \frac{1}{2}x - \frac{\sqrt{65x}}{2}$ **b)** $y = -3x + \sqrt{50}$, $y = -3x - \sqrt{50}$

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1. $x + y - 40 = 0$ **2. a)** $\sqrt{450}$ **b)** $\sqrt{450}$ **3.** The points are the same distance from, and on opposite sides of, the diagonal. **4.** (0, 10) and (30, 40); (10, 10) and (30, 30); (10, 20) and (20, 30) The points in each pair are the same distance from, and on opposite sides of, the diagonal.