5.3 Investigation: Graphing Quadratic Functions by Factoring

If a quadratic equation of the form $ax^2 + bx + c = 0$ can be solved by factoring, then the x-intercepts of the quadratic function $y = ax^2 + bx + c$ can be found. The coordinates of the vertex can be deduced from the x-intercepts.

1. The x-intercepts of a quadratic function are the x-coordinates of the points where the graph crosses the x-axis, or where $y = 0$. For the function $y = x^2 - 2x - 8$, let $y = 0$ and solve the resulting quadratic equation by factoring.

2. What are the x-intercepts?

3. On a grid, plot the points where the graph crosses the x-axis.

4. Communication The axis of symmetry of the graph of $y = x^2 - 2x - 8$ passes through the vertex. Use symmetry to find the x-coordinate of the vertex. Explain your reasoning.

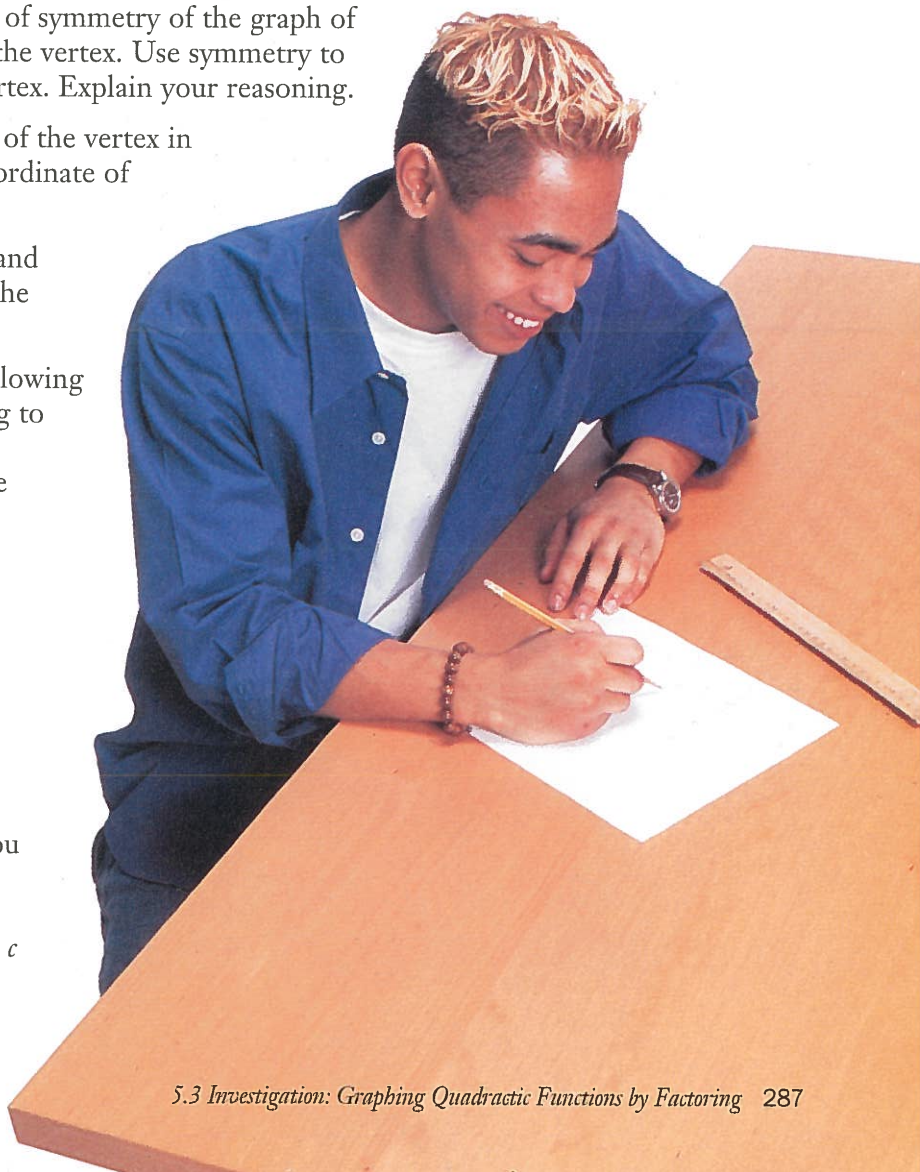
5. Substitute the x-coordinate of the vertex in $y = x^2 - 2x - 8$ to find the y-coordinate of the vertex.

6. Plot the vertex on the grid and draw a smooth curve through the three points.

7. Sketch the graphs of the following quadratic functions by factoring to find the x-intercepts, and then deducing the coordinates of the vertex.

a) $y = x^2 - 8x + 12$
b) $y = x^2 - 2x - 3$
c) $y = x^2 + 10x + 21$
d) $y = x^2 + 4x$
e) $y = x^2 - 6x + 5$
f) $y = x^2 - 6x - 7$
g) $y = x^2 - 8x$
h) $y = x^2 - 2x - 35$

8. Communication Could you use the above method for sketching the graph of $y = ax^2 + bx + c$ when $ax^2 + bx + c$ is a perfect square? Explain.



5 or –5 **19.** $w = 1$ m, $l = 18$ m **20.** 9 **21.** $w = \frac{8}{3}$ m, $l = 6$ m **22.** 8 cm, 15 cm **23.** 11 **24.** 8 cm
25. a) Outer: 8 cm × 8 cm, Inner: 4 cm × 4 cm
b) Outer: 9 cm × 9 cm, Inner: 3 cm × 3 cm **26.** 0.5 m
27. a) $w = 70$ mm, $l = 135$ mm **b)** The widths are the same but a Canadian $20 bill is longer with length 152 mm. **28. a)** $x = -3$; 1 root **b)** $x = -3$; there are two roots that are equal. **29. a)** $x = -\frac{1}{6}$ or $x = 1$ **b)** $x = -\frac{3}{4}$ or $x = 1$ **30. a)** $x = 0$ or $x = 1$ **b)** $x = -\frac{1}{2}$ or $x = 2$ **c)** $k = -4$ or $k = 2$ **d)** $z = \frac{1}{2}$ **e)** $x = -4$ or $x = 3$ **f)** $g = \frac{8}{3}$ or $g = 6$ **g)** $y = 3$ or $y = -3$ **h)** $n = 1$ or $n = -1$ **i)** $x = 0$ or $x = 7$ **31. a)** $x = -y$ or $x = -4y$ **b)** $x = -\frac{y}{2}$ or $x = 3y$ **c)** $x = \frac{y}{2}$ **d)** $x = \frac{y}{4}$ or $x = -2y$ **e)** $x = 0$ or $x = -\frac{y}{5}$ **f)** $x = 0$ or $x = \frac{7y}{3}$ **32. a)** $x^2 - x - 6 = 0$
b) Yes, any constant multiple of $x^2 - x - 6 = 0$
33. a) $x^2 + (-p - q)x + pq = 0$ **34. a)** 10 **b)** $-\frac{1}{3}$

Career Connection p. 286

1. a) $16x^2 = 144$ m^2 $\Rightarrow x^2 = 9$ m^2 $\Rightarrow x = 3$m **b)** 2 m

Modelling Math p. 286

a) $-5t^2 + 9t + 2 = 0$ **b)** $(5t + 1)(-t + 2) = 0$; 2 s

Section 5.3 p. 287

1. $x = 4$ or $x = -2$ **2.** 4, –2 **4.** $x = 1$ **5.** $y = -9$
7. a) x–intercepts: 2, 6; vertex: (4, –4)
b) x–intercepts: –1, 3; vertex: (1, –4)
c) x–intercepts: –7, –3; vertex: (–5, –4)
d) x–intercepts: –4, 0; vertex: (–2, –4)
e) x–intercepts: 1, 5; vertex: (3, –4)
f) x–intercepts: –1, 7; vertex: (4, –15)
g) x–intercepts: 0, 8; vertex: (4, –16)
h) x–intercepts: –5, 7; vertex: (1, –36) **8.** No, because $ax^2 + bx + c$ is a perfect square, there is only one x – intercept, which is also the vertex. A point on each side of the vertex is needed to complete the sketch.

Section 5.4 pp. 292–295

Practice 1. a) $x = -1$ or $x = -5$ **b)** $x = -4$ or $x = 2$
c) $x = -1$ or $x = 3$ **d)** $x = 5$ or $x = 7$ **e)** $x = -2$ **f)** $y = 1$
2. a) $x = \frac{1}{2}$ or $x = 1$ **b)** $x = -\frac{1}{5}$ or $x = 3$ **c)** $x = -\frac{3}{2}$ or $x = 4$ **d)** $x = \frac{1}{3}$ **e)** $x = -\frac{3}{2}$ or $x = \frac{3}{4}$ **f)** $x = -\frac{1}{2}$ or $x = \frac{2}{3}$ **g)** $x = \pm\frac{3}{2}$ **h)** $x = -\frac{3}{2}$ or $x = -\frac{5}{2}$ **i)** $x = 0$ or $x = \frac{5}{2}$ **j)** $x = -\frac{5}{3}$ or $x = -2$ **3. a)** $x = \frac{-5 \pm \sqrt{17}}{2}$; $x = -0.44$ or $x = -4.56$

b) $x = \frac{3 \pm \sqrt{13}}{2}$; $x = 3.30$ or $x = -0.30$ **c)** $x = \frac{1 \pm \sqrt{13}}{2}$; $x = 2.30$ or $x = -1.30$ **d)** $x = \frac{-7 \pm \sqrt{41}}{2}$; $x = -0.30$ or $x = -6.70$ **e)** $x = \frac{5 \pm \sqrt{33}}{2}$; $x = 5.37$ or $x = -0.37$

f) $z = \frac{1 \pm \sqrt{17}}{2}$; $z = 2.56$ or $z = -1.56$

g) $x = \frac{-1 \pm \sqrt{29}}{2}$; $x = 2.19$ or $x = -3.19$

h) $a = \frac{1 \pm \sqrt{21}}{2}$; $a = 2.79$ or $a = -1.79$ **i)** $x = \frac{-3 \pm \sqrt{65}}{4}$; $x = 1.27$ or $x = -2.77$ **j)** $x = \frac{1 \pm \sqrt{13}}{6}$; $x = 0.77$ or $x = -0.43$ **k)** $x = \frac{-1 \pm \sqrt{41}}{4}$; $x = 1.35$ or $x = -1.85$

l) $x = \frac{-3 \pm \sqrt{21}}{-6}$; $x = -0.26$ or $x = 1.26$

Applications and Problem Solving 4. a) 2.25 m **b)** 3.8 s
5. a) $x = 0$ or $x = 1.6$ **b)** $x = 1.3$ or $x = -0.3$ **c)** $x = 3.4$ or $x = -1.4$ **d)** $c = 3.9$ or $c = -0.9$ **e)** $n = 6.6$ or $n = -0.6$ **f)** $x = 2.8$ or $x = -1.3$ **g)** $x = -0.3$ or $x = -2$ **h)** $x = 10.7$ or $x = 1.3$ **i)** $d = 6.2$ or $d = 0.8$ **j)** $g = 0$ or $g = 1.9$
6. a) $x = 0.65$ or $x = -4.65$ **b)** $x = 5.16$ or $x = -1.16$
c) $y = 1.31$ or $y = -0.13$ **d)** $n = 1.67$ or $n = -0.5$
e) $x = -0.04$ or $x = -1.05$ **f)** $a = 9.94$ or $a = -12.44$
7. 82 m **8.** $w = 56$ m, $l = 116$ m **9.** 12 cm, 16 cm
10. a) 7.5 cm **b)** 6562.5 cm^3 **11.** $l = 8.1$ m, $w = 6.1$ m
12. 5.6 m **13.** $b = 7.4$ cm, $h = 5.4$ cm **14.** $h = 23.7$ cm, $w = 29.7$ cm **15.** 13 cm × 8 cm **16. a)** between 17 cm and 18 cm **b)** 17.72 cm **17. a)** $N = 110$ jackets; $P = \$160$ **b)** $N = 130$ jackets; $P = \$120$ **18.** 3.58 units
19. a) 6; 10 **b)** $s = \frac{p(p-1)}{2}$ **c)** $p = 11$ **d)** no
20. $b^2 - 4ac \geq 0$ **21.** $b^2 - 4ac = 0$ **22.** 3.2 cm
23. a) no real solutions **b)** two real, equal roots
c) two real, distinct roots **d)** two real, distinct,