REVIEW OF *Key* CONCEPTS

2.1 Length of a Line Segment

Refer to the Key Concepts on page 71.

To find the exact length of the line segment joining (4, 6) and (2, –3), and the approximate length, to the nearest tenth, use the length formula.

$$\begin{aligned} l &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2-4)^2 + (-3-6)^2} \\ &= \sqrt{(-2)^2 + (-9)^2} \\ &= \sqrt{85} \\ &\doteq 9.2 \end{aligned}$$

The exact length is $\sqrt{85}$, and the approximate length is 9.2, to the nearest tenth.

To determine the radius of a circle with centre (0, 0) and equation $x^2 + y^2 = 81$, compare it to the general equation $x^2 + y^2 = r^2$. In the equation $x^2 + y^2 = 81$, $r^2 = 81$, so $r = 9$.
The radius of a circle with centre (0, 0) and equation $x^2 + y^2 = 81$ is 9.

1. Find the exact length of the line segment joining each pair of points.
a) A(7, 9) and B(1, 1) **b)** W(4, 5) and X(–2, 3)
c) E(–2, 8) and F(–5, 5) **d)** R(–10, 5) and T(4, –1)
e) U(1.2, –0.4) and V(–0.8, 3.6)

2. Write an equation for the circle with centre (0, 0) and the given radius.
a) radius 12 **b)** radius 20

3. The following equations model circles with centre (0, 0). Determine the radius of each circle. Round to the nearest tenth, if necessary.
a) $x^2 + y^2 = 121$ **b)** $x^2 + y^2 = 20$
c) $x^2 + y^2 = 400$ **d)** $x^2 + y^2 = 0.49$

4. **Measurement** △ABC has vertices A(4, 5), B(–1, 2), and C(5, 1).
a) Classify the triangle by side length.
b) Determine the perimeter of the triangle, to the nearest tenth.

5. **Measurement** Determine the perimeter of the quadrilateral with vertices E(2, 4), F(–2, 3), G(–3, –2), and H(3, –4), to the nearest tenth.

6. **Measurement** Verify that D(0, 3), E(–1, –6), and F(4, –2) are the vertices of an isosceles triangle.

7. **Measurement** **a)** Verify that the quadrilateral with vertices P(3, 3), Q(0, 1), R(3, –1), and S(6, 1) is a rhombus.
b) Determine the exact perimeter of the rhombus.

2.2–2.3 Midpoint of a Line Segment

Refer to the Key Concepts on page 77.

To determine the coordinates of the midpoint of the line segment with endpoints A(−4, 9) and B(6, 1), use the formula for midpoint.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{-4+6}{2}, \frac{9+1}{2}\right)$$
$$=(1, 5)$$

The coordinates of the midpoint are (1, 5).

8. Find the coordinates of the midpoint of each line segment.

a) P(2, −7) and Q(−3, 5)

b) S(6, −2) and T(2, 2)

c) M(2, −5) and N(5, −1)

d) G(−2, 0) and H(−4, 3)

e) V(2.9, 3.2) and W(3.1, −4.2)

f) $A\left(3\frac{1}{2}, \frac{1}{2}\right)$ and $B\left(-2\frac{1}{2}, 1\frac{1}{2}\right)$

9. For a line segment KL, one endpoint is K(5,1) and the midpoint is M(1, 4). Find the coordinates of endpoint L.

10. **Measurement** △ABC has vertices A(1, 4), B(−3, −2), and C(3, 0). Find the lengths of the three medians. Express each length as

a) an exact value **b)** an approximate value, to the nearest tenth

11. **Measurement** Quadrilateral PQRS has vertices P(0, 7), Q(−2, 1), R(4, −1), and S(6, 3). A second quadrilateral is formed by joining the midpoints of the sides of quadrilateral PQRS. Show that the opposite sides of the second quadrilateral are equal.

12. Verify that the diagonals of the rectangle with vertices A(−4, 1), B(−1, 3), C(3, −3), and D(0, −5) bisect each other.

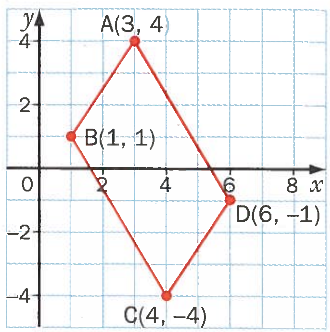
2.4 Verifying Properties of Geometric Figures

Refer to the Key Concepts on page 95.

To verify that the quadrilateral with vertices A(3, 4), B(1, 1), C(4, −4), and D(6, −1) is a parallelogram, calculate the slopes of the sides.

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m_{AB}=\frac{4-1}{3-1}=\frac{3}{2} \qquad m_{DC}=\frac{-1-(-4)}{6-4}=\frac{3}{2} \qquad m_{BC}=\frac{-4-1}{4-1}=-\frac{5}{3} \qquad m_{AD}=\frac{4-(-1)}{3-6}=-\frac{5}{3}$$



Since the slopes of opposite sides are equal, opposite sides are parallel. The quadrilateral is a parallelogram.

13. Verify that a quadrilateral with vertices K(2, –5), L(7, –3), M(1, 4), and N(–4, 2) is a parallelogram.

14. A square has vertices at U(–2, 1), V(2, 3), W(4, –1), and X(0, –3). Verify that the diagonals perpendicularly bisect each other.

15. Given △DEF with vertices D(–4, –1), E(4, 3), and F(0, –5), verify that
a) △DEF is isosceles
b) the line segment joining the midpoints of the equal sides is parallel to the third side and half the length of the third side

16. Quadrilateral RSTU has vertices R(2, 4), S(–2, 2), T(–1, 0), and U(3, 2). Verify that
a) quadrilateral RSTU is a rectangle
b) the diagonals of the rectangle bisect each other and are equal in length

17. Quadrilateral ABCD has vertices A(3, 4), B(–1, 2), C(–3, –4), and D(5, –6). Verify that
a) the quadrilateral formed by joining the midpoints of the sides of quadrilateral ABCD is a rhombus
b) the diagonals of the rhombus bisect each other at right angles

18. △DEF has vertices D(4, 5), E(–4, 3), and F(0, –5). Determine
a) an equation of EB, the median from E to DF
b) an equation of FA, the altitude from F to DE
c) an equation of PQ, the right bisector of EF

19. △ABC has vertices A(4, 2), B(0, 4), and C(–2, –2). Determine the coordinates of the circumcentre of △ABC.

20. △PQR has vertices P(1, 3), Q(–1, –1), and R(5, 1). Determine the coordinates of the centroid of △ABC. PQR

21. Determine the equation of the right bisector of the line segment joining C(–3, 5) and D(3, 2).

22. The equation of a circle with centre O(0, 0) is $x^2 + y^2 = 20$. The points P(2, –4) and Q(4, 2) are the endpoints of chord PQ. AB right bisects the chord PQ at C. Verify that the centre of the circle lies on the right bisector of chord PQ.

2.5 Distance From a Point to a Line

Refer to the Key Concepts on page 103.

To find the distance from the point P(1, 0) to the line $x - 2y + 4 = 0$, to the nearest tenth, first write the equation $x - 2y + 4 = 0$ in the slope and y-intercept form to find the slope of the line.

$$y = \frac{1}{2}x + 2$$

The slope of the line is $\frac{1}{2}$.

So, the slope of the perpendicular from P(1, 0) to $x - 2y + 4 = 0$ is -2.
To find the equation of the perpendicular, use the point-slope form, where $(x_1, y_1) = (1, 0)$ and $m = -2$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= -2(x - 1) \\ y &= -2x + 2 \\ 2x + y &= 2 \end{aligned}$$

Then, solve the following system of equations to find the point of intersection.

$$\begin{aligned} x - 2y + 4 &= 0 \qquad (1) \\ 2x + y &= 2 \qquad (2) \end{aligned}$$

The point of intersection is (0, 2).
Finally, use the distance formula to find the distance from (0, 2) to (1, 0).
The distance from the point P(1, 0) to the line $x - 2y + 4 = 0$ is 2.2, to the nearest tenth.

23. Find the shortest distance from the origin to each line. Round to the nearest tenth, if necessary.

a) $y = x + 5$
b) $y = -2x + 1$
c) $y = \frac{1}{3}x - 4$
d) $x + y = 8$
e) $x + 2y = 10$
f) $x + 2y + 2 = 0$

24. Find the shortest distance from the given point to the given line. Round to the nearest tenth, if necessary.

a) (1, 4) and $y = x - 5$
b) (−2, 2) and $2x + y = 8$
c) (−1, −3) and $x + 3y - 9 = 0$

25. Find the distance from the point P(−3, 6) to the line $x = 4$.

26. Find the distance from the point Q(4, −3) to the line $y = 2$.

27. **Measurement** △RST has vertices R(−3, 4), S(−1, 0), and T(3, 2).

a) Find the length of the altitude from S to RT.
b) Find the area of the triangle.

Chapter Test

1. Find the exact distance between each pair of points.
a) (4, –1) and (1, 3)
b) (2, 0) and (4, –5)

2. Write an equation for the circle with centre (0, 0) and the given radius.
a) radius 3
b) radius 7

3 The following are equations of circles with centre (0, 0). Determine the exact radius of each circle.
a) $x^2 + y^2 = 100$
b) $x^2 + y^2 = 15$

4. **Measurement** The endpoints of a diameter of a circle are (–1, –3) and (3, 6). Calculate the radius of the circle, to the nearest tenth.

5. **Measurement** The vertices of a right triangle are A(3, 3), B(–1, 4), and C(1, –5). Find the area of the triangle.

6. **Measurement** The vertices of a triangle are R(3, 0), S(–1, 3), and T(0, –2).
a) Classify the triangle by side length.
b) Find the perimeter of the triangle, to the nearest tenth.

7. Determine the coordinates of the midpoint of the line segment with endpoints (–2, –5) and (–8, 3).

8. For the line segment DE, one endpoint is D(3, 1) and the midpoint is M(0, –4). Find the coordinates of endpoint E.

9. **Measurement** $\triangle$DEF is a right triangle with vertices D(–2, 5), E(–4, 1), and F(2, 3). Verify that the midpoint of the hypotenuse is equidistant from the three vertices of the triangle.

10. Verify that the vertices A(–6, 1), B(2, –5), C(6, 1), and D(2, 4) are the vertices of a trapezoid.

11. Quadrilateral RSTU has vertices R(3, 2), S(0, 4), T(–2, 1), and U(1, –1). Verify that
a) quadrilateral RSTU is a square
b) the diagonals of quadrilateral RSTU perpendicularly bisect each other and are equal in length

12. The midpoints of the sides of quadrilateral RSTU, with vertices R(–4, 1), S(–2, 5), T(2, 1), and U(2, –7), are joined to form a quadrilateral. Verify that the quadrilateral formed is a parallelogram.

13. Determine the equation of the right bisector of the line segment joining E(2, 6) and F(4, –2).

14. $\triangle ABC$ has vertices A(6, 3), B(2, 5), and C(0, 1). Determine the coordinates of the circumcentre of $\triangle ABC$.

15. Find the shortest distance from the origin to each line. Round to the nearest tenth, if necessary.

a) $y = x + 3$

b) $2x - y + 7 = 0$

16. Find the shortest distance from the point (3, 1) to the line $3x + y = -2$, to the nearest tenth.

17. The equation of a circle with centre O(0, 0) is $x^2 + y^2 = 40$. The points A(–2, 6) and B(–6, –2) are endpoints of chord AB. DE right bisects chord AB at F.

a) Verify that the centre of the circle lies on the right bisector of chord AB.

b) Find the distance from the centre of the circle to chord AB, to the nearest tenth.

Achievement Check

1 2 3 4

The longest crude-oil pipeline in the world is the one from Edmonton, Alberta, to Buffalo, New York, a distance of 2858 km. In one region, the pipeline follows the path given by $y = 2x + 20$, where each unit on the grid represents 1 km. A town in that region is centred at (50, 5) and has radius 5 km. New by-laws require that the pipeline not be within 50 km of an urban area. How close does the pipeline come to the town? Does the pipeline need to be rerouted? If so, what is the minimum length of the existing pipeline that must be rerouted? Explain and justify your reasoning.

Technology Extension p. 106

1 Equations of Lines **5. a)** $x + y = 7$ **b)** $3x + 4y = -6$ **c)** $5y = 4x + 4$ **d)** $y = x - 5$

Rich Problem p. 111

2. Five locations at (6, 9), (6, 8), (7, 7), (7, 8), and (8, 7).

Review of Key Concepts pp. 112–115

1. a) 10 **b)** $\sqrt{40}$ **c)** $\sqrt{18}$ **d)** $\sqrt{232}$ **e)** $\sqrt{20}$ **2. a)** $x^2 + y^2 = 144$ **b)** $x^2 + y^2 = 400$ **3. a)** 11 **b)** 4.5 **c)** 20 **d)** 0.7 **4. a)** scalene **b)** 16.0 **5.** 23.6 **6.** $DE = \sqrt{82}$, $EF = \sqrt{41}$, $FD = \sqrt{41}$; $EF = FD$; so $\triangle DEF$ is isosceles. **7. a)** $PQ = \sqrt{13}$, $QR = \sqrt{13}$, $RS = \sqrt{13}$, $SP = \sqrt{13}$ **b)** $4\sqrt{13}$ **8. a)** $\left(-\frac{1}{2}, -1\right)$ **b)** (4, 0) **c)** $\left(\frac{7}{2}, -3\right)$ **d)** $(-3, \frac{3}{2})$ **e)** (3, −0.5) **f)** $(\frac{1}{2}, 1)$ **9.** (−3, 7) **10. a)** from A: $\sqrt{26}$, from B: $\sqrt{41}$, from C: $\sqrt{17}$ **b)** 5.1, 6.4, 4.1 **11.** Two sides have length $\sqrt{17}$; two sides have length $\sqrt{20}$. **12.** Their midpoints coincide at $\left(-\frac{1}{2}, -1\right)$. **13.** Opposite sides are parallel (two sides have slope $\frac{2}{5}$ and two have slope $-\frac{7}{6}$). **14.** Their midpoints coincide at (1, 0), and one slope is 3 and the other is $-\frac{1}{3}$. **15. a)** $ED = EF = \sqrt{80}$ and so $\triangle DEF$ is isosceles. **b)** The line segment joining the midpoints of the equal sides has slope −1 and length $\sqrt{8}$ or about 2.83, and the third side has slope −1 and length $\sqrt{32}$ or about 5.66. **16. a)** Opposite sides are parallel and adjacent sides are perpendicular (two sides have slope −2 and two have slope $\frac{1}{2}$). **b)** Each diagonal has midpoint (0.5, 2) and so they bisect each other. Each diagonal has length 5. **17. a)** Opposite sides of the quadrilateral formed from the midpoints are parallel (two sides have slope $\frac{4}{3}$ and two have slope $-\frac{4}{3}$) and all sides have length 5. **b)** One diagonal is vertical and the other is horizontal; so they are perpendicular. Their midpoints coincide at (1, −1); so they bisect each other. **18. a)** $x + 2y - 2 = 0$ **b)** $4x + y + 5 = 0$ **c)** $x - 2y = 0$ **19.** $\left(\frac{5}{7}, \frac{3}{7}\right)$ **20.** $\left(\frac{5}{3}, 1\right)$ **21.** $4x - 2y + 7 = 0$ **22.** The right bisector of PQ has equation $x + 3y = 0$, which passes through the origin. **23. a)** 3.5 **b)** 0.4 **c)** 3.8 **d)** 5.7 **e)** 4.5 **f)** 0.9 **24. a)** 5.7 **b)** 4.5 **c)** 6.0 **25.** 7 **26.** 5 **27. a)** $\sqrt{10}$ **b)** 10

Chapter Test pp. 116–117

1. a) 5 **b)** $\sqrt{29}$ **2. a)** $x^2 + y^2 = 9$ **b)** $x^2 + y^2 = 49$ **3. a)** 10 **b)** $\sqrt{15}$ **4.** 4.9 **5.** 17 **6. a)** scalene **b)** 13.7 **7.** (−5, −1) **8.** (−3, −9) **9.** The midpoint is $\sqrt{10}$ units from each vertex. **10.** AB and DC both have slope $-\frac{3}{4}$, and the slopes of BC and AD are different. **11. a)** Opposite sides are parallel and adjacent sides are perpendicular (two have slope $\frac{3}{2}$ and two have slope $-\frac{2}{3}$) and all sides have length $\sqrt{13}$. **b)** The midpoints of the diagonals coincide at $(\frac{1}{2}, \frac{3}{2})$; so they bisect each other. Their slopes are −5 and $\frac{1}{5}$; so they are perpendicular. Each diagonal has length $\sqrt{26}$. **12.** Opposite sides of the quadrilateral formed by joining the midpoints are parallel (two sides have slope 0 and two have slope −3). **13.** $x - 4y + 5 = 0$ **14.** (3, 2) **15. a)** 2.1 **b)** 3.1 **16.** 3.8 **17. b)** 4.5

Problem Solving p. 119

Applications and Problem Solving **1. a)** 90 **b)** 160 **c)** 204 **d)** 385 **2.** 987 **3. a)** $y = x + 4$; 13, 16, 25 **b)** $y = 2x + 1$; 17, 12, 101 **c)** $x + y = 9$; 3, 8, 0 **d)** $y = x - 7$; 5, 2, 7 **e)** $y = x^2 + 1$; 6, 401, 10 001 **f)** $y = 4x - 1$; 39, 16, 119 **4. a)** 6 **b)** 3 **5. a)** 156 **b)** 1260 **6. a)** 41, 82, 81 **b)** 42, 68, 110 **c)** 37, 50, 65 **7.** A: $p - q$, −11, 5, 3; B: $q - p$, 11, −5, −3; C: $2p + q$, −4, −2, −12; D: $p^2 - q^2$, −11, −15, −27; E: $pq + 1$, −29, −3, 19 **8. a)** 146 **b)** 21 316 **9. a)** 6 **b)** 4

Problem Solving p. 122

2. 32 **3.** G, P, W; R **4. a)** It is about 1.25 times the surface area of Pluto, assuming both bodies are spherical. **b)** about 540 times **5.** 43 **6. a)** 1 **b)** 2 **8.** L-shape or Z-shape **9. b)** The square is being rotated 90° counterclockwise. **c)** same as square 3 **d)** same as square 1 **10.** $\frac{1}{57}$ **11. a)** 10 **b)** 10

Cumulative Review p. 123

Chapter 1 **1. a)** (−14, −41) **b)** (0.5, −3) **2. a)** (−1, 1) **b)** (1, 0) **3. a)** (2, 2) **b)** infinitely many solutions **c)** (−3, −4) **d)** $(0, \frac{1}{2})$ **4.** 600 g portobello, 400 g oyster