REVIEW OF *Key* CONCEPTS

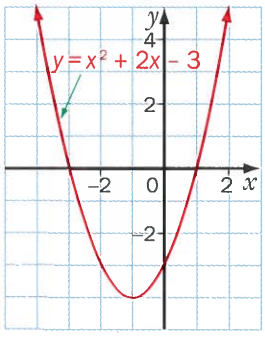
5.1 Solving Quadratic Equations by Graphing

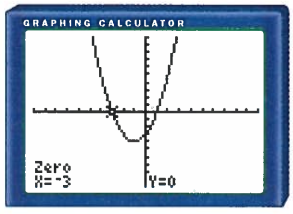
Refer to the Key Concepts on page 275.

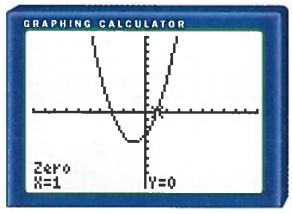
To solve $x^2 + 2x - 3 = 0$ by graphing, graph the related quadratic function $y = x^2 + 2x - 3$ using paper and pencil, a graphing calculator, or graphing software.

Find the values of the x-intercepts.

x	y
2	5
1	0
0	-3
-1	-4
-2	-3
-3	0
-4	5







The graph intersects the x-axis at (1, 0) and (–3, 0).
The roots of the equation $x^2 + 2x - 3 = 0$ are 1 and –3.

1. Solve by graphing.
a) $x^2 - 2x - 3 = 0$
b) $x^2 + x - 2 = 0$
c) $x^2 - 9 = 0$
d) $-x^2 - 3x + 4 = 0$
e) $x^2 - 2x + 1 = 0$
f) $x^2 + 6x = -5$
g) $x^2 = 5x$

2. Solve graphically using a graphing calculator or graphing software. Round each answer to the nearest tenth, if necessary.
a) $2x^2 + x - 3 = 0$
b) $-3x^2 + 7x - 2 = 0$
c) $-x^2 + 2 = 3x$

3. Measurement The length of a rectangle is 2 cm more than the width. The area is 24 cm^2. What are the dimensions of the rectangle?

Refer to the Key Concepts on page 282.

To solve $x^2 - 3x = 4$ by factoring, first write the equation in the form $ax^2 + bx + c = 0$.

$$x^2 - 3x - 4 = 0$$

Factor the left side: $(x - 4)(x + 1) = 0$

Use the zero product property: $x - 4 = 0$ or $x + 1 = 0$

$x = 4$ or $x = -1$

The roots are 4 and -1.

4. Solve by factoring.

a) $x^2 + 3x - 28 = 0$

b) $y^2 - 5y + 6 = 0$

c) $g^2 + 7g + 10 = 0$

d) $x^2 + 8x + 16 = 0$

e) $2x^2 - 7x - 30 = 0$

f) $9x^2 - 4 = 0$

g) $2x^2 + 5x = 3$

h) $2n^2 = 27 - 15n$

i) $8k^2 - 3k = 0$

j) $18 + m^2 = 82$

k) $\frac{x^2}{6} - x + \frac{4}{3} = 0$

l) $\frac{x^2}{2} - \frac{x}{2} = -\frac{1}{8}$

5. Curling The playing surface in the game of curling is a rectangular sheet of ice with an area of about 225 m^2. The width is about 40 m less than the length. Find the approximate dimensions of the playing surface.

5.3 Graphing Quadratic Functions by Factoring

The coordinates of the vertex and two other points can be used to sketch the graph of a quadratic function. The coordinates can be found by factoring.

To sketch the graph of $y = x^2 - 4x - 5$, solve $x^2 - 4x - 5 = 0$ to find the x-intercepts.

$$x^2 - 4x - 5 = 0$$
$$(x - 5)(x + 1) = 0$$
$$x - 5 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 5 \quad \text{or} \quad x = -1$$

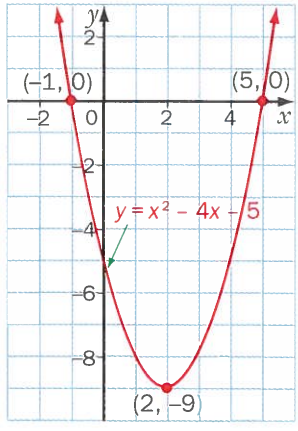
The x-intercepts are 5 and -1.

Plot the points $(5, 0)$ and $(-1, 0)$.
Since the two x-intercepts are reflection images of each other in the axis of symmetry, the x-coordinate of the vertex is 2.
Substitute 2 for x in $y = x^2 - 4x - 5$ to find the y-coordinate of the vertex.

$$y = x^2 - 4x - 5$$
$$= 2^2 - 4(2) - 5$$
$$= -9$$

The coordinates of the vertex are $(2, -9)$.

Plot the vertex on the grid and draw a smooth curve through the three points.



6. Sketch the graphs of the following quadratic functions by factoring to locate the x-intercepts, and then finding the coordinates of the vertex.

a) $y = x^2 + 6x + 8$ **b)** $y = x^2 - 2x - 15$
c) $y = x^2 - 8x$ **d)** $y = x^2 + 2x - 8$
e) $y = x^2 - 4x - 5$ **f)** $y = x^2 - 4x + 3$

5.4 The Quadratic Formula

Refer to the Key Concepts on page 292.

To use the quadratic formula to solve $2x^2 + 5x - 12 = 0$, substitute $a = 2$, $b = 5$, and $c = -12$ into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-5 \pm \sqrt{5^2 - 4(2)(-12)}}{2(2)}$$
$$= \frac{-5 \pm \sqrt{25 + 96}}{4}$$
$$= \frac{-5 \pm \sqrt{121}}{4}$$
$$= \frac{-5 \pm 11}{4}$$

So, $x = \frac{-5+11}{4}$ or $x = \frac{-5-11}{4}$

$= \frac{6}{4}$ $= -\frac{16}{4}$

$= \frac{3}{2}$ $= -4$

The roots are $\frac{3}{2}$ and -4.

7. Solve using the quadratic formula.

a) $x^2 - 7x + 12 = 0$ **b)** $x^2 - 3x + 5 = 0$

c) $n^2 + n - 42 = 0$ **d)** $x^2 - 1 = 0$

e) $2x^2 + 3x = 0$ **f)** $4x^2 - 12x + 9 = 0$

g) $7g^2 + 2 = 9g$ **h)** $4m^2 = 3 + 4m$

8. Solve using the quadratic formula. Express answers as exact roots and as approximate roots, to the nearest hundredth.

a) $x^2 + 5x - 3 = 0$ **b)** $k^2 - 9k = 1$

c) $8w^2 = 2 - 3w$ **d)** $3 = 5x + 3x^2$

9. Solve, to the nearest hundredth.

a) $6x^2 + x - 4 = 0$ **b)** $1.2x^2 + 0.5x - 0.3 = 0$

10. Measurement A rectangle has a perimeter of 46 cm and an area of 120 cm^2. Find its dimensions by writing an equation and using the quadratic formula to solve it.

irrational roots **24. a)** $x = -\frac{y}{2}$ or $x = -8y$

b) $x = (-1 \pm \sqrt{2})y$

Modelling Math p. 295

a) 4.0 s **b)** 2 s

Technology Extension: Solving Quadratic Equations pp. 296–297

1 Using a Graphing Calculator Program **1.** Line
i) Displays "ENTER COEFFICIENTS".
ii) Prompts you to enter the values of A, B, C
iii) Calculates $B^2 - 4AC$ and assigns the value to D.
iv) Determines if $D \geq 0$. v) If $D \geq 0$, the program continues to line vi. vi) Displays $\frac{-B+\sqrt{D}}{2A}$, $\frac{-B-\sqrt{D}}{2A}$. vii) If $D < 0$, the program continues to line viii. viii) Displays "NO REAL SOLUTIONS". **2. a)** no real solutions **b)** $x = \frac{1}{4}$ or $x = 2$ **c)** $x = -\frac{1}{2}$ or $x = -1$ **d)** $x = 0$ or $x = \frac{4}{5}$ **e)** $x = 0.1$ or $x = -3.5$ **f)** $x = -2$ or $x = 3$ **3. a)** $x = 1.65$ or $x = -6.65$ **b)** no real solutions **c)** $x = 0$ or $x = 0.64$

2 Using Preprogrammed Calculators **1. a)** $x = -1$ or $x = 4$ **b)** $x = -5$ or $x = 2$ **c)** $x = -1$ or $x = -5$ **d)** $y = 1$ or $y = 6$ **e)** $x = 8$ **f)** $n = 0.7$ or $n = -0.7$ **g)** $x = -1$ or $x = \frac{1}{2}$ **h)** $p = -\frac{2}{3}$ or $p = \frac{3}{2}$ **i)** $x = 0$ or $x = -\frac{3}{7}$ **j)** $x = -2$ or $x = -\frac{1}{3}$ **k)** $x = 3$ or $x = -5$ **l)** $s = 1$ or $s = -4$ **m)** $x = -\frac{4}{3}$ or $x = 2$ **n)** $x = -8$ or $x = \frac{1}{2}$ **o)** $z = -3.3$ or $z = 3$ **2. a)** $x = 1.32$ or $x = -5.32$ **b)** $a = 5.24$ or $a = 0.76$ **c)** $x = 4.56$ or $x = 0.44$ **d)** $x = 1.26$ or $x = -4.76$ **e)** $t = -0.23$ or $t = -1.43$ **f)** $x = 5.42$ or $x = -1.42$ **g)** $k = 1.34$ or $k = -5.47$ **h)** $x = 3.85$ or $x = -2.85$ **i)** $x = 1.41$ or $x = -1.21$ **j)** $w = 1.24$ or $w = -3.24$ **k)** $x = 2.65$ or $x = 0.15$ **l)** $b = -5.88$ or $b = 1.21$ **3. a – b)** There are no real roots since $(2)^2 - 4(1)(13) < 0$.

Rich Problem: The Golden Ratio pp. 298–300

1 Determining ϕ **1. a)** $\phi = \frac{1+\sqrt{5}}{2}$ **b)** $\phi = 1.618$

2 Geometry and ϕ **1.** Given: AB = 2 and E bisects AB. This gives AE = EB = 1. Also, ABCD is a square. So, AB = BC = CD = AD = 2. $(EC)^2 = (EB)^2 + (BC)^2$ by Pythagorean theorem. Thus, $(EC)^2 = 1^2 + 2^2 = 5$. Therefore, $EC = \sqrt{5}$ and $EF = \sqrt{5}$. $AF = AE + EF = 1 + \sqrt{5}$. Since AB = 2, $\frac{AF}{AB} = \frac{1+\sqrt{5}}{2} = \phi$. **2.** The base of a rectangle ABCD is bisected at E. Diagonals are drawn from C to E and D to E, forming an isosceles triangle EFD. If AB = 2 and AC = $\sqrt{2\sqrt{5}+5}$, then show $\frac{CE}{CD} = \phi$.

3 Fibonacci Numbers and ϕ **1.** 377, 610 **2. a)** $\frac{5}{3}$ **b)** $\frac{13}{8}$ **c)** The Fibonacci Sequence is $a_1 = a_2 = 1$, $a_{n+2} = a_n + a_{n+1}$. If $r_n = \frac{a_{n+1}}{a_n}$, then $r_{n+1} = 1 + \frac{1}{r_n}$

d) $$\frac{34}{21} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1+1}}}}}}$$

3. a) 2, 5, 7, 12, 19, 31, 50, 81, 131, 212 **b)** $\frac{5}{2} = 2.5$, $\frac{7}{5} = 1.4$, $\frac{12}{7} = 1.714\,285\,714$, $\frac{19}{12} = 1.58\overline{3}$, $\frac{31}{9} = 1.631\,578\,947$, $\frac{50}{31} = 1.612\,903\,226$, $\frac{81}{50} = 1.62$, $\frac{131}{81} = 1.617\,283\,951$, $\frac{212}{131} = 1.618\,320\,611$. The ratios converge to ϕ.

4 ϕ in Architecture, Design, and Nature **1.** 18 m
2. slant height = 185.852 091 7 m;
$\frac{\text{slant height}}{115\text{ m}} = 1.616\,105\,145 \doteq \phi$
3. AC = 167.343 957 2 mm, BC = 103.324 730 8 mm;
$\frac{AB}{BC} = 1.619\,592\,482 \doteq \phi$

Review of Key Concepts pp. 301–304

1. a) $x = -1$ or $x = 3$ **b)** $x = -2$ or $x = 1$ **c)** $x = -3$ or $x = 3$ **d)** $x = -4$ or $x = 1$ **e)** $x = 1$ **f)** $x = -1$ or $x = -5$ **g)** $x = 0$ or $x = 5$ **2. a)** $x = -1.5$ or $x = 1$ **b)** $x = 0.3$ or $x = 2$ **c)** $x = -3.6$ or $x = 0.6$ **3.** $w = 4$ cm, $l = 6$ cm **4. a)** $x = -7$ or $x = 4$ **b)** $y = 2$ or $y = 3$ **c)** $g = -2$ or $g = -5$ **d)** $x = -4$ **e)** $x = -\frac{5}{2}$ or $x = 6$ **f)** $x = -\frac{2}{3}$ or $x = \frac{2}{3}$ **g)** $x = \frac{1}{2}$ or $x = -3$ **h)** $n = \frac{3}{2}$ or $n = -9$ **i)** $k = 0$ or

$k = \frac{3}{8}$ **j)** $m = -8$ or $m = 8$ **k)** $x = 2$ or $x = 4$ **l)** $x = \frac{1}{2}$

5. $l = 45$ m, $w = 5$ m **6. a)** x-intercepts: $-2, -4$; vertex $(-3, -1)$ **b)** x-intercepts: 5, -3; vertex $(1, -16)$ **c)** x-intercepts: 0, 8; vertex $(4, -16)$ **d)** x-intercepts: -4, 2; vertex $(-1, -9)$ **e)** x-intercepts: 5, -1; vertex $(2, -9)$ **f)** x-intercepts: 1, 3; vertex $(2, -1)$ **7. a)** $x = 4$ or $x = 3$ **b)** no real roots **c)** $n = 6$ or $n = -7$ **d)** $x = 1$ or $x = -1$ **e)** $x = 0$ or $x = -\frac{3}{2}$ **f)** $x = \frac{3}{2}$ **g)** $g = 1$ or $g = \frac{2}{7}$ **h)** $x = \frac{3}{2}$ or $x = -\frac{1}{2}$ **8. a)** $x = \frac{-5 \pm \sqrt{37}}{2}$; $x = 0.54$ or $x = -5.54$ **b)** $k = \frac{9 \pm \sqrt{85}}{2}$; $k = 9.11$ or $k = -0.11$

c) $w = \frac{-3 \pm \sqrt{73}}{16}$; $w = 0.35$ or $w = -0.72$

d) $x = \frac{-5 \pm \sqrt{61}}{6}$; $x = 0.47$ or $x = -2.14$

9. a) $x = 0.74$ or $x = -0.90$ **b)** $x = 0.33$ or $x = -0.75$
10. 8 cm × 15 cm

Chapter Test p. 305

1. a) $x = -4$ or $x = 2$ **b)** $x = \pm 3$ **c)** $x = 2$ **d)** $x = 0$ or $x = -6$ **e)** no real roots **f)** $x = 1$ **2. a)** $q = -5$ or $q = 3$ **b)** $w = 3$ or $w = 8$ **c)** $k = \frac{1}{2}$ or $k = -4$ **d)** $x = -\frac{1}{3}$ or $x = \frac{2}{3}$ **e)** $x = 0$ or $x = \frac{3}{2}$ **f)** $x = \pm\frac{1}{2}$ **g)** $x = -1$ **h)** $x = \pm 12$ **i)** $x = \frac{1}{2}$ or $x = 1$ **3. a)** x – intercepts: $-1, -5$; vertex $(-3, -4)$ **b)** x – intercepts: 1, -5; vertex $(-2, -9)$ **4. a)** $y = 6$ or $y = -3$ **b)** $x = 4$ **c)** $x = -2$ **d)** $x = \frac{17}{3}$ or $x = 0$ **e)** $x = \pm\frac{2}{3}$ **f)** no real roots **g)** $w = \frac{1}{6}$ or $w = -\frac{2}{3}$ **h)** $d = \frac{3}{4}$ or $d = -\frac{1}{3}$ **i)** $p = -\frac{2}{5}$ or $p = -1$ **5. a)** $x = \frac{-5 \pm \sqrt{53}}{2}$; $x = 1.14$ or $x = -6.14$ **b)** $t = \frac{-9 \pm \sqrt{33}}{8}$; $t = -0.41$ or $t = -1.84$

c) $x = \frac{1 \pm \sqrt{85}}{6}$; $x = 1.70$ or $x = -1.37$ **6.** $w = 13$ cm, $l = 21$ cm **7.** 3.5 m

Problem Solving: Solve a Simpler Problem p. 307

Applications and Problem Solving **1.** 70 **2.** 20
3. 32 square units **4.** 22.5 square units
5. 44.5 square units **6. a)** 253 square units
b) 398 units **7. a)** 40 **b)** $n(n + 3)$ **c)** 2650 **d)** 15
e) $\frac{n(n+1)}{2}$ **f)** 1275

Problem Solving: Use a Table or Spreadsheet p. 310

Applications and Problem Solving **1. a)**

Month Position	Profit ($)	Net Financial ($)
1	−2100	−2100
2	−1900	−4000
3	−1700	−5700
4	−1500	−7200
5	−1300	−8500
6	−1100	−9600
7	−900	−10 500
8	−700	−11 200
9	−500	−11 700
10	−300	−12 000
11	−100	−12 100
12	100	−12 000
13	300	−11 700
14	500	−11 200
15	700	−10 500
16	900	−9600
17	1100	−8500
18	1300	−7200
19	1500	−5700
20	1700	−4000
21	1900	−2100
22	2100	0
23	2300	2300
24	2500	4800

b) month 23 **c)** $n = 100(m - 11)^2 - 12\,100$
2. 5 months sooner **3.** 13 **4. a)** 276 **b)** 440
5. a)

Time (s)	Height (m)
1	44.1
2	78.4
3	102.9
4	117.6
5	122.5
6	117.6
7	102.9
8	78.4
9	44.1
10	0

b) 5 s **c)** 122.5 m **d)** $h = -4.9(t - 5)^2 + 122.5$
e) Domain: $0 \le t \le 10$; Range: $0 \le h \le 122.5$ **6.** 84
7. 10 **8.** No, he will not save more than $1000.