

3.3. * Common Factoring *

↳ aka Common Dividing

Warmup: 1. Expand $(2x-3y)^3$

$$\begin{aligned} \text{Sol'n} &= (2x-3y)(2x-3y)(2x-3y) \\ &= (4x^2 - 6xy - 6xy + 9y^2)(2x-3y) \\ &= (4x^2 - 12xy + 9y^2)(2x-3y) \end{aligned}$$

$$\begin{aligned} &= 8x^3 - 12x^2y - 24x^2y + 36xy^2 + 18xy^2 - 27y^3 \\ &= 8x^3 - 36x^2y + 54xy^2 - 27y^3, \text{ collected likes} \end{aligned}$$

Remark: Factor implies building brackets. Factor means find a divider of every term in the given polynomial. Factors are divisors aka dividers.

Ex₁ Factor a) 8

$$\begin{aligned} \text{Sol'n} &= 4(2), \text{ factor} \\ &= 2(2)(2), \text{ fully factored.} \end{aligned}$$

b) $9xy^2$

$$\begin{aligned} \text{Sol'n} &= 3(3)xy^2, \text{ factor} \\ &= 3(3)(x)(y)(y), \text{ fully factored.} \end{aligned}$$

Ex₂ Factor a) $12x+8$

↑
means change expression to a product of its "dividers"

[Not sure that is a word :)]

$$= 2(6x+4), \text{ good!}$$

$$= 4(3x+2), \text{ best!!}$$

↑ "other factor"

"∵ 4 is bigger than 2,

∴ 4 is called "Greatest common factor aka divider"

b) $15x+9x^3$

$$= 3x(5+3x^2) \checkmark$$

↑ ↑
Greatest Common Number Factor
Greatest shared letter Factor

c) $30m^2n - 45mn + 15mn^2$

Asks: Biggest number you can cleanly divide 30, 45, and 15 by? Biggest power of m and n you can divide from all 3 terms in given trinomial? Ans: $15mn$.

So, G.C.F. = $15mn$ to the front.

$$= 15mn(2m-3+1n) \checkmark$$

Note: You can always physically or mentally check your factoring using your expanding skills from yesterday.

Watch: Checking Ex₂ a) $4(3x+2)$
 $= 12x + 8 \checkmark$

b) $3x(5+3x^2)$
 $= 15x + 9x^3 \checkmark$

c) $15mn(2m-3+1n)$
 $= 30m^2n - 45mn + 15mn^2 \checkmark$

So, you can always check your work : [A side: In the past, students have submitted quizzes and tests, I ask, "How did you do?" They answer, "Perfect," I answer, "How do you know?" They answer, "I checked them all :"]

Remark₂: Sometimes "like dividers" aka common factors can be polynomials.

Ex₃ Factor.

a) $2x(x+4) - 3(x+4)$

Sol'n = $2x(x+4) - 3(x+4)$, think of binomial $(x+4)$ as the like

✓ = $(x+4)(2x-3)$, group to front
G.C.F. divider

Sol'n₂ Let $k = (x+4)$

So, $2x(x+4) - 3(x+4)$

= $2x(k) - 3(k)$, sub out
= $k(2x-3)$

✓ = $(x+4)(2x-3)$, sub back in.

b) $4m(x+5) + 5(x+5)$

Sol'n = $(x+5)(4m+5)$, group to front
DONE ✓

c) $7x^2(x+1) - 1(x+1)$

= $(x+1)(7x^2-1)$ Don't forget the 1.

Group to front ✓

Ex₄ Group Factor. [Look for a common divider for all the terms. If none exists, look for GROUPS that have a common divider.]

a) $6am - 2bm - 9an + 3bn$

Sol'n "Find 2 groups (often pairs of terms) that have a unique common factor."

= $6am - 2bm - 9an + 3bn$

= $2m(3a-b) - 3n(3a-b)$, common factor both teams aka groups

= $(3a-b)(2m-3n)$, binomial to the front.

b) $4m - 14n^2 - 6mn + 21n^3$

Sol'n Reorder to make 2 groups aka teams that each have their own unique divider.

= $4m - 6mn - 14n^2 + 21n^3$

= $2m(2-3n) - 7n^2(2-3n)$, C.F. 2x

= $(2-3n)(2m-7n^2)$ DONE ✓

↑ other "2-3n to factor the front"

p 150 # (1-4) every other letter

#5

#7.