Practice

1. Write an equation in standard form for the line that passes through the given point and has the given slope.

a) $(3, 5)$; $m = 2$ **b)** $(4, 2)$; $m = 5$ **c)** $(-6, 6)$; $m = -3$

d) $(-2, -1)$; $m = -4$ **e)** $(2, -8)$; $m = \frac{1}{2}$ **f)** $(1, -3)$; $m = -\frac{1}{2}$

g) $(4, -\frac{1}{2})$; $m = 1$ **h)** $(-\frac{1}{3}, 3)$; $m = \frac{3}{2}$ **i)** $(-1, -1)$; $m = -1.5$

2. Write an equation in standard form for the line through the given points.

a) A(4, 5) and B(3, 7) **b)** C(−2, 9) and D(1, 0)

c) E(−3, −6) and F(−5, −7) **d)** G(−2, −3) and H(3, −1)

e) J(−2, 3) and K(2, 8) **f)** L(0, 4) and M(−3, 0)

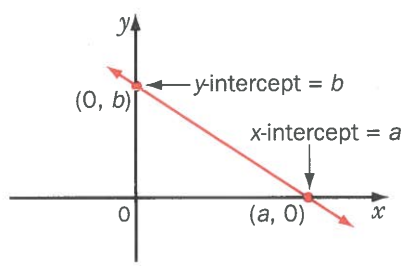
3. Write an equation in standard form for the horizontal line and the vertical line through each point.

a) $(4, 5)$ **b)** $(-3, 2)$ **c)** $(-5, -6)$ **d)** $\left(\frac{1}{2}, 8\right)$

e) $\left(9, -\frac{1}{3}\right)$ **f)** $(0, -9)$ **g)** $(-1, 0)$ **h)** $(0, 0)$

2 Using the Slope and *y*-Intercept Form

The equation $y = mx + b$ is the **slope and *y*-intercept form** of a line. The slope is m, and the *y*-intercept is b.



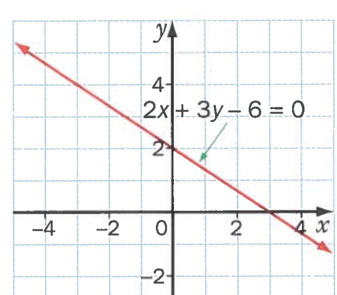
Example 1 Finding the Slope and *y*-Intercept Given an Equation

Find the slope and *y*-intercept of the line $2x + 3y - 6 = 0$.

Solution

Rewrite the equation in the form $y = mx + b$.

$$2x + 3y - 6 = 0$$
$$3y = -2x + 6$$
$$\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3}$$
$$y = -\frac{2}{3}x + 2$$



The slope is $-\frac{2}{3}$ and the *y*-intercept is 2.

Example 2 Finding the Slope and y-Intercept Given Two Points

Find the slope and y-intercept of the line through the points F(3, –1) and G(5, 7).

Solution

To write an equation in the form $y = mx + b$, first find the slope using the given points.

$$\begin{aligned} m_{FG} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{7 - (-1)}{5 - 3} \\ &= \frac{8}{2} \\ &= 4 \end{aligned}$$

Use the point-slope form and either of the known points for (x_1, y_1).
Using F(3, –1) for (x_1, y_1),

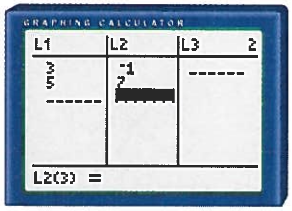
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-1) &= 4(x - 3) \\ y + 1 &= 4(x - 3) \\ y + 1 &= 4x - 12 \\ y &= 4x - 13 \end{aligned}$$

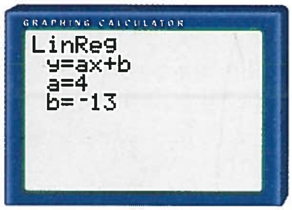
For $y = mx + b$, m is the slope and b is the y-intercept.
For $y = 4x - 13$, the slope is 4 and the y-intercept is –13.

So, the line that passes through the points F(3, –1) and G(5, 7) has a slope of 4 and a y-intercept of –13.

Technology Extension

To find an equation using a graphing calculator, input the coordinates of the two given points in two lists using the STAT EDIT menu. Then, use the LinReg (linear regression) instruction to find an equation for the line.





Practice

1. Find the slope and y-intercept of each line.

a) $y = 3x + 4$
b) $4x + y - 6 = 0$
c) $x - y + 5 = 0$
d) $x - 2y = 8$
e) $3x + 6y - 8 = 0$
f) $0 = x - 4y - 4$
g) $5x - 2y - 6 = 0$
h) $2x + 3y = 0$
i) $10x + 0.5y - 3 = 0$

2. Find the slope and y-intercept of the line that passes through the following pairs of points.

a) (2, 4) and (4, 6)
b) (1, 2) and (–2, 5)
c) (–3, –1) and (1, 7)
d) (4, –2) and (5, –5)
e) (–2, –2) and (0, –1)
f) (2, 0) and (4, –1)
g) (–2, –1) and (–3, –5)
h) (3, 2) and (–2, 3)
i) (7, 4) and (1, 2)

Example 2 Perpendicular Lines

Write an equation of the line perpendicular to $3x + y - 6 = 0$ and passing through the point P(5, 2).

Solution

To use the point-slope form, you need a point on the line and the slope of the line. The slope of the new line is the negative reciprocal of the slope of $3x + y - 6 = 0$, because the lines are perpendicular.

To find the slope of $3x + y - 6 = 0$, write the equation in the form $y = mx + b$.

$$3x + y - 6 = 0$$
$$y = -3x + 6$$

The slope of $3x + y - 6 = 0$ is -3.

The slope, m, of a line perpendicular to $3x + y - 6 = 0$ is the negative reciprocal of -3, which is $\frac{1}{3}$.

A point on the line is P(5, 2).

Use the point-slope form:	$y - y_1 = m(x - x_1)$
Substitute known values:	$y - 2 = \frac{1}{3}(x - 5)$
Expand:	$y - 2 = \frac{1}{3}x - \frac{5}{3}$
Multiply both sides by 3:	$3y - 6 = x - 5$
Write in standard form:	$0 = x - 3y + 1$

An equation of the line perpendicular to $3x + y - 6 = 0$ and passing through the point P(5, 2) is $x - 3y + 1 = 0$.

Practice

1. Determine an equation for each of the following lines.

a) the line parallel to $y = 3x + 4$ and passing through the point (2, 1)

b) the line parallel to $2x - y = 7$ and passing through the point (–3, 2)

c) the line parallel to $x + 2y - 5 = 0$ and passing through the point (5, –3)

d) the line parallel to $3x - 9y - 1 = 0$ and having the same y-intercept as $2x + y - 8 = 0$

2. Determine an equation for each of the following lines.

a) the line perpendicular to $y = -2x + 4$ and passing through the point (4, 6)

b) the line perpendicular to $x - 3y - 1 = 0$ and passing through the point (1, –1)

c) the line perpendicular to $x + 2y - 8 = 0$ and passing through the point (–3, –4)

d) the line perpendicular to $4x - 2y + 3 = 0$ and having the same x-intercept as $2x + 3y - 10 = 0$

Technology Extension p. 81

1 Length of a Line Segment **1.** names the program; clears the memory; prompts user for x_1-data and reads it into variable P; prompts user for y_1-data and reads it into variable Q; prompts user for x_2-data and reads it into variable R; prompts user for y_2-data and reads it into variable S; assigns the difference R – P to the variable X; assigns the difference S – Q to the variable Y; assigns the square root of the sum of the squares of X and Y to the variable L; prints "LENGTH IS" and prints the value of L. **2. a)** 6.4 **b)** 17.1 **c)** 27.33

2 Midpoint of a Line Segment **1.** Change line 6 to (R + P)/2 → X. Change line 7 to (S + Q)/2 → Y. Delete line 8. Change the last line to DISP"MIDPOINT IS". Add lines DISP"X=",X and DISP"Y=",Y. **2. a)** (–1, 6) **b)** (2.5, –3.5) **c)** (0.3, –3.45)

3 Collinear Points **2. a)** collinear **b)** not collinear **3.** Answers may vary. **a)** (7, 6) **b)** (–7, 6)

Review: Equations of Lines pp. 85–87

1 Using the Point-Slope Form
1. a) $2x - y - 1 = 0$ **b)** $5x - y - 18 = 0$ **c)** $3x + y + 12 = 0$ **d)** $4x + y + 9 = 0$ **e)** $x - 2y - 18 = 0$ **f)** $x + 2y + 5 = 0$ **g)** $2x - 2y - 9 = 0$ **h)** $3x - 2y + 7 = 0$ **i)** $3x + 2y + 5 = 0$ **2. a)** $2x + y - 13 = 0$ **b)** $3x + y - 3 = 0$ **c)** $x - 2y - 9 = 0$ **d)** $2x - 5y - 11 = 0$ **e)** $5x - 4y + 22 = 0$ **f)** $4x - 3y + 12 = 0$ **3. a)** $y - 5 = 0$; $x - 4 = 0$ **b)** $y - 2 = 0$; $x + 3 = 0$ **c)** $y + 6 = 0$; $x + 5 = 0$ **d)** $y - 8 = 0$; $2x - 1 = 0$ **e)** $3y + 1 = 0$; $x - 9 = 0$ **f)** $y + 9 = 0$; $x = 0$ **g)** $y = 0$; $x + 1 = 0$ **h)** $y = 0$; $x = 0$

2 Using the Slope and *y*-Intercept Form
1. a) 3, 4 **b)** –4, 6 **c)** 1, 5 **d)** $\frac{1}{2}$, –4 **e)** $-\frac{1}{2}$, $\frac{4}{3}$ **f)** $\frac{1}{4}$, –1 **g)** $\frac{5}{2}$, –3 **h)** $-\frac{2}{3}$, 0 **i)** –20, 6 **2. a)** 1, 2 **b)** –1, 3 **c)** 2, 5 **d)** –3, 10 **e)** $\frac{1}{2}$, –1 **f)** $-\frac{1}{2}$, 1 **g)** 4, 7 **h)** $-\frac{1}{5}$, $\frac{13}{5}$ **i)** $\frac{1}{3}$, $\frac{5}{3}$

3 Parallel and Perpendicular Lines
1. a) $3x - y - 5 = 0$ **b)** $2x - y + 8 = 0$ **c)** $x + 2y + 1 = 0$ **d)** $x - 3y + 24 = 0$ **2. a)** $x - 2y + 8 = 0$ **b)** $3x + y - 2 = 0$ **c)** $2x - y + 2 = 0$ **d)** $x + 2y - 5 = 0$

Section 2.4 pp. 95–99

Practice **1.** PQ and OR have slope $\frac{1}{5}$; OP and RQ have slope $\frac{5}{3}$ **2.** XY has slope $\frac{2}{3}$; XZ has slope $-\frac{3}{2}$; the slopes are negative reciprocals, so XY is perpendicular to XZ **3. a)** Both segments have slope $-\frac{4}{3}$ and so are parallel. **b)** PQ = 5, KM = 10; PQ = $\frac{1}{2}$KM **4.** Opposite sides are parallel (two have slope 0 and two have slope $-\frac{4}{3}$) and all sides have length 5; so PQRS is a rhombus. **5. a)** KL and NM have slope –1; KN and LM have slope 1. Thus, opposite sides are parallel and adjacent sides are perpendicular. KLMN is a rectangle. **b)** KM and LN both have length $\sqrt{26}$ **6.** $x + y - 5 = 0$ **7. a)** The midpoint of BC is (4, 3). The line containing (4, 3) and A(3, 4) has slope –1. BC has slope 1. Thus, A(3, 4) is on the perpendicular bisector of BC.

b) The midpoint of QR is (0, 0). The line containing (0, 0) and P(1, 3) has slope 3. QR has slope $-\frac{1}{3}$. Thus, P(1, 3) is on the perpendicular bisector of QR. **c)** The midpoint of LM is (2, –2). The line containing (2, –2) and K(–2, –4) has slope $\frac{1}{2}$. LM has slope –2. Thus, K(–2, –4) is on the perpendicular bisector of LM. **8.** The midpoint of CD is (2, –1). The line containing (2, –1) and O(0, 0) has slope $-\frac{1}{2}$. CD has slope 2. Thus, O(0, 0) is on the right bisector of CD. **9.** Slope AC is $\frac{1}{5}$ and slope BD is –5. The slopes are negative reciprocals, so the diagonals AC and BD are perpendicular.

10. The opposite sides PQ and OR both have slope $\frac{1}{5}$ and so are parallel; the other two sides have different slopes, so are not parallel. OPRQ is a trapezoid.

11. PS and QR both have slope $\frac{2}{3}$; RS and QP are both vertical. Thus, opposite sides are parallel and PQRS is a parallelogram. **12. a)** KL has slope 1, KM has slope –1. KL and KM are perpendicular. **b)** The midpoint of LM is a distance of $\sqrt{17}$ from each vertex. **13.** Opposite sides of the quadrilateral are parallel (two sides have slope $\frac{2}{5}$ and two have slope –5) and so the quadrilateral is a parallelogram. **14. a)** $7x + 2y + 1 = 0$ **b)** $x - y + 1 = 0$ **c)** $x + 4y - 2 = 0$ **15. a)** $y = 0$, $x - 4 = 0$, $x + y - 4 = 0$ **b)** All three medians intersect at (4, 0). **16.** (4, 0) **17.** (2, 2)
Applications and Problem Solving **18.** The vertices of the triangle are A(2, 0), B(–4, 3), and C(–1, –6). AB = AC = $\sqrt{45}$; AC has slope 2 and AB has slope $-\frac{1}{2}$