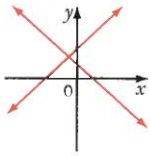
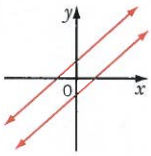
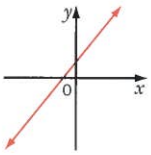
The graphs of two linear equations in two variables may intersect at one point, be parallel and distinct, or coincide.

Graphs of Lines	Slopes of Lines	Intercepts	Number of Solutions
Intersecting 	Different	Different unless the lines intersect on one axis or at the origin	One
Parallel and distinct 	Same	Different	None
Coincident 	Same	Same	Infinitely many

Example 5 Analyzing Systems

Analyze each system to determine whether the system has one solution, no solution, or infinitely many solutions.

a) $2x + y = 6$ (1)
$y - 8 = -2x$ (2)

b) $3x + y = 1$ (1)
$6x + 2y = 2$ (2)

c) $2x + y - 4 = 0$ (1)
$x + 2y - 6 = 0$ (2)

Solution

Before analyzing each system, express both equations in the form $y = mx + b$.

a) $y = -2x + 6$ (1)
$y = -2x + 8$ (2)

Both lines have a slope of -2. Therefore, the lines are either parallel and distinct or they coincide. Since the y-intercepts are different, the lines must be parallel and distinct.

The system has no solution.

b) $y = -3x + 1$ (1)
$y = -3x + 1$ (2)

Both lines have the same slope, -3, and the same y-intercept, 1. The two lines coincide.

The system has infinitely many solutions.

c) $y = -2x + 4$ (1)
$y = -\frac{1}{2}x + 3$ (2)

The slopes of the lines are -2 and $-\frac{1}{2}$. The lines are not parallel and distinct, and they do not coincide. They intersect at a single point.

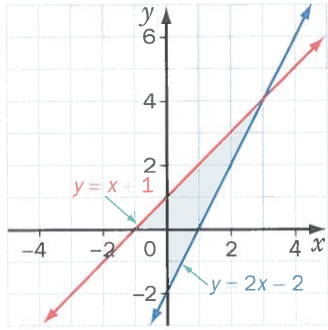
The system has one solution.

Key Concepts

1. To solve a system of linear equations graphically,
   a) graph the equations using a graphing calculator, graphing software, or paper and pencil
   b) determine the coordinates of the point of intersection
   c) check the solution by substituting it in each of the original equations
2. The number of solutions to a linear system is
   a) exactly one, if the lines intersect
   b) none, if the lines are parallel and distinct
   c) infinitely many, if the lines coincide

Communicate Your Understanding

1. The linear system $y = x + 1$ and $y = 2x - 2$ is modelled graphically at the right. State the solution to this system. Justify your answer.



2. Describe how you would solve the linear system $2x + 3y = 3$ and $3x - y = 7$ graphically.

3. Explain why a system of linear equations cannot have exactly two solutions.

4. Decide how many solutions there are to the following linear system just by looking at the equations. Explain your reasoning.

$$y = 7x - 4$$
$$y = 7x + 5$$

Practice

A

1. Solve each system by graphing. Check your solutions.

a) $y = x - 1$, $y = 9 - x$ **b)** $y = x + 3$, $y = 1 - x$ **c)** $y = 2x + 1$, $y = x - 2$ **d)** $y = 1 - 2x$, $y = x - 5$

2. Solve each system by graphing. Check your solutions.

a) $x - y = -5$, $x + y = 1$ **b)** $5x - 2y = 10$, $x + 2y = 2$ **c)** $3x - 2y = 12$, $x - 2y = 8$ **d)** $2x + 3y = -12$, $2x - y = -4$

3. Solve each system by graphing. Check your solutions.

a) $x - y = 4$, $x + y = 2$ **b)** $x + y = 5$, $x - y = -7$ **c)** $x + 2y = 2$, $x + y = 3$ **d)** $x + 3y = -1$, $2x + 6y + 2 = 0$

e) $2x + y = 12$, $3x - 2y = 18$ **f)** $2x + y = -2$, $4x = y - 16$ **g)** $y = 2x - 3$, $2x - y = 5$ **h)** $2x + y = -5$, $3x - y = -5$

i) $2x - y = 5$, $y = x - 3$ **j)** $3x + y = -11$, $y = 2x + 4$ **k)** $3x + 4y - 16 = 0$, $x - 2y - 2 = 0$ **l)** $3x = y + 8$, $6x - 2y - 1 = 0$

m) $2x + 3y = 7$
$2x - 3y = 13$

n) $y = \frac{1}{2}x + 3$
$x = 2y - 6$

o) $2x - 3y = 4$
$3x - 4y = 5$

p) $3x + 2y - 10 = 0$
$2x - 3y + 2 = 5$

4. Solve by graphing. Check each solution.

a) $y = 4x$
$y = 2x + 1$

b) $2x - 2y - 1 = 0$
$x - 4y + 4 = 0$

c) $x + 2y = 0$
$x - 2y = -2$

d) $x + y = -1$
$3x - y = 7$

5. Solve by graphing.

a) $3x + 2y = 3$
$2x + 10y = -5$

b) $x + 2y = 10$
$x - y = 5$

c) $2x + 3y - 7 = 0$
$3x - 5y - 13 = 0$

d) $y = -0.5x - 1$
$y = 0.25x + 1$

e) $y = 3$
$y = 2.58x - 3$

f) $y = 0.35x + 6.02$
$y = -3.22x - 3.12$

6. Without graphing, determine whether each system has one solution, no solution, or infinitely many solutions.

a) $2x + y = 5$
$4x + y = 9$

b) $3x - y = 0$
$6x - 2y = 3$

c) $x + y = 2$
$3x = 6 - 3y$

d) $x + 4y = 8$
$y + 2x = 0$

e) $2y = 3x - 1$
$8y - 4 = 12x$

f) $2y - x - 4 = 0$
$3x - 6y - 12 = 0$

Applications and Problem Solving

7. Geography The total number of states in Austria and Germany is 25. Germany has 7 more states than Austria. Solve the following system of equations graphically to find the number of states in each country.

$$a + g = 25$$
$$g = a + 7$$

B

8. Health clubs Phoenix Health Club charges a $200 initiation fee, plus $15 a month. Champion Health Club charges a $100 initiation fee, plus $20 a month. The costs can be compared using the following equations.

Phoenix Cost: $C = 200 + 15m$
Champion Cost: $C = 100 + 20m$

a) Find the point of intersection of the two lines.
b) After how many months are the costs the same?
c) If you joined a club for only a year, which club would be less expensive?

9. Coordinate geometry The arms of an angle lie on the lines $y = \frac{2}{3}x + 7$ and $3x + 2y = -12$. What are the coordinates of the vertex of the angle?

10. Coordinate geometry The three lines $y = 2x$, $y = 6 - x$, and $y = -2$ intersect to form a triangle. What are the coordinates of the vertices of the triangle?

11. Coordinate geometry The three lines $y = \frac{1}{3}x - 2$, $x - y = 4$, and $x + 3y = 4$ intersect to form a triangle. Find the coordinates of the vertices.

12. **Geometry** Name the type of quadrilateral formed when the lines $x - y = -3$, $y = x - 2$, $y = -\frac{1}{2}x + 5$, and $x + 2y + 12 = 0$ intersect.

C

13. Write an equation that forms a system of equations with $x + y = 4$, so that the system has

a) no solution **b)** infinitely many solutions **c)** one solution

14. Write a system of equations that has the point (3, 2) as

a) the only solution **b)** one of infinitely many solutions

15. **Communication** If (0, 3) and (2, 4) are both solutions to a system of two linear equations, does the system have any other solutions? Explain.

16. Sketch a graph to represent a system of two equations with one solution, so that the two lines have

a) different x-intercepts and different y-intercepts

b) the same x-intercept but different y-intercepts

c) different x-intercepts but the same y-intercept

d) the same x-intercept and the same y-intercept

17. **Graphing calculator** For many systems solved graphically using a graphing calculator, the point of intersection does not fall within the standard viewing window.

a) Solve each of the following systems using a graphing calculator.

$y = -2x - 16$
$y = 4x + 59$

$y = x - 24$
$y = -2x + 120$

$y = x - 2$
$y = \frac{x}{2} - 10$

b) **Communication** Describe how you found suitable values for the window variables in each case.

c) Compare your answers to part b) with those of your classmates.

Modelling Math Comparing Costs and Revenues

To raise money for a school reunion, students sell T-shirts. The cost of the T-shirts includes an \$800 design and set-up charge, plus \$4 per T-shirt. The T-shirts sell for \$20 each. The cost and the revenue can be modelled by the following system of equations.

Dollar Cost: $d = 800 + 4t$

Dollar Revenue: $d = 20t$

a) Solve the system graphically.

b) The solution shows the break-even point, at which the cost and revenue are equal. How many T-shirts must the students sell to break even?

c) Suppose the students lose money. How many T-shirts are sold?

d) Suppose the students make a profit. How many T-shirts are sold?

Answers

Chapter 1

Getting Started p. 2

1. 10; The check digit will be 0. **2. a)** 9 **b)** 9 **c)** 2 **d)** 8 **3. a)** No, the check digit should be 6. **b)** Yes, the check digit is correct. **c)** Yes, the check digit is correct. **4.** Answers may vary. 123 456 717; 223 456 740 **5. a)** $10 - m$ **b)** 0 **c)** The check digit is equal to $10 - m$ if $m \neq 0$ and 0 if $m = 0$.

Review of Prerequisite Skills p. 3

1. a) $x + 2$ **b)** $2x + 8$ **c)** $3y - 5$ **d)** $-5a + 3$ **e)** $6x + 14$ **f)** $5z - 8$ **g)** $7t + 41$ **h)** $2x - 9$ **2. a)** $6x$ **b)** $-2c$ **c)** x **d)** $3n$ **e)** $x + 2y$ **f)** $3p - r$ **3. a)** 8 **b)** 2 **c)** -6 **d)** -5 **4. a)** 7 **b)** -3 **c)** 2 **d)** 12 **e)** $-\frac{1}{2}$ **f)** $\frac{3}{2}$ **g)** 4 **h)** -5 **i)** $-\frac{5}{2}$ **j)** -4 **k)** -1 **l)** 2 **5. a)** $x = 11 - 3y$ **b)** $x = 5y - 8$ **c)** $x = 2y - 4$ **d)** $x = \frac{5 - 3y}{2}$ **6. a)** $y = 3 - 2x$ **b)** $y = x - 2$ **c)** $y = \frac{-1 - 2x}{4}$ **d)** $y = \frac{3x - 4}{2}$ **10. a)** (3, 1) **b)** (5, −2) **c)** (−1, 6) **d)** (4, 8) **e)** (−4, −5) **f)** (2, −1) **11. a)** $9x - 4y + 1$ **b)** $13m^2 - 6m - 19$ **c)** $-a - 3b - 10$ **d)** $-e - 2$ **12. a)** $x - 8y + 10$ **b)** $-t^2 - 5t - 11$ **c)** $-9a + 3b + 1$ **d)** $12e - 1$

Section 1.1 pp. 4–5

1 Ordered Pairs and One Equation **1. a)** (1, 13), (24, −10) **b)** (−2, −4), (−12, 0) **c)** (2, 3) **d)** (0.5, −2.5) **2. a)** 3, 9, 10, −2 **b)** 2, −9, 11, −2 **c)** −1, 5, 13, 10 **d)** 5, 3, −4, −7

2 Ordered Pairs and Two Equations **1. a)** (1, 2) **b)** (−3, 1) **c)** (2, 3) **d)** (6, −8) **e)** (−2, −5) **f)** (−4, 7) **2. a)** (4, −3) **b)** (6, 3) **c)** (−1, 0) **d)** Answers may vary. (0, 0)

3 Problem Solving **1. a)** 55 **b)** 3 days **c)** $55 **2. a)** The equations represent the same graph. **b)** Answers may vary. (−1, 2), (−2, 3) **3.** The equations represent parallel and distinct lines. The lines never intersect.

Section 1.2 pp. 12–14

Practice **1. a)** (5, 4) **b)** (−1, 2) **c)** (−3, −5) **d)** (2, −3) **2. a)** (−2, 3) **b)** (2, 0) **c)** (2, −3) **d)** (−3, −2) **3. a)** (3, −1) **b)** (−1, 6) **c)** (4, −1) **d)** infinitely many solutions **e)** (6, 0) **f)** (−3, 4) **g)** no solution **h)** (−2, −1) **i)** (2, −1) **j)** (−3, −2) **k)** (4, 1) **l)** no solution **m)** (5, −1) **n)** infinitely many solutions **o)** (−1, −2) **p)** (2, 2) **4. a)** (0.5, 2) **b)** (2, 1.5) **c)** (−1, 0.5) **d)** (1.5, −2.5) **5. a)** (1.5, −0.8) **b)** (6.7, 1.7) **c)** (3.9, −0.3) **d)** (−2.7, 0.3) **e)** (2.3, 3) **f)** (−2.6, 5.1) **6. a)** one solution **b)** no solution **c)** infinitely many solutions **d)** one solution **e)** no solution **f)** no solution **7.** Austria: 9, Germany: 16 **8. a)** (20, 500) **b)** 20 months **c)** Champion **9.** (−6, 3) **10.** (2, 4), (−1, −2), (8, −2) **11.** $(3, -1), (5, -\frac{1}{3}), (4, 0)$ **12.** parallelogram **13.** Answers may vary. **a)** $x + y = 5$ **b)** $2x + 2y = 8$ **c)** $x + 2y = 4$ **14.** Answers may vary. **a)** $x + y = 5$, $x - y = 1$ **b)** $x - y = 1$, $2x - 2y = 2$ **15.** The system has infinitely many solutions: all points on the line $x - 2y + 6 = 0$. **17. a)** (−12.5, 9); (48, 24); (−16, −18)

Modelling Math p. 14

a) $(t, d) = (50, 1000)$ **b)** 50 **c)** less than 50 **d)** greater than 50

Career Connection p. 15

1. south: 5000, north: 125 000

Section 1.3 pp. 21–23

Practice **1. a)** $x = 8 - 3y$ **b)** $x = -4y - 13$ **c)** $x = 7y + 7$ **d)** $x = 2y - 1$ **2. a)** $y = 11 - 6x$ **b)** $y = -5x - 9$ **c)** $y = x + 2$ **d)** $y = 3x + 4$ **3. a)** (2, 2) **b)** (−1, 1) **c)** (2, −1) **d)** (−2, −3) **e)** (3, 0) **f)** (3, 2) **g)** (4, −5) **h)** (5, 0) **i)** (−2, 3) **j)** (−2, −2) **k)** (−1, 1) **l)** (−3, −4) **m)** (1, 0) **n)** (1, 3) **o)** no solution **p)** (3, −1) **q)** infinitely many solutions **r)** (−1, −5) **s)** no solution **t)** (1, 1) **u)** (−1, 1) **4. a)** $(\frac{1}{2}, -1)$ **b)** $(\frac{7}{11}, -\frac{1}{11})$ **c)** $(3, -\frac{6}{5})$ **d)** $(1, -\frac{1}{3})$ **e)** $(-1, \frac{2}{7})$