2.8 *Solving Log Equations*

Warmup: 1. Evaluate $y = \log_3 44$ to 2 decimal places.

Sol'n,

$$y = \frac{\log 44}{\log 3}, \text{ change of base formula}$$

$$y \doteq 3.44$$

Remark₁: Sometimes an exponential equation that is difficult to solve becomes easier when manipulated to log format. (Yesterday's lesson.)

ex Solve $2^x = 7.1$

Remark₂: Sometimes a log equation that is difficult to solve becomes easier when manipulated to exponential form aka "delogged" form. (Today.)

Ex₁ Solve. a) $5.5 = \log\left(\frac{x}{1.9}\right) + 3.2$

Strategy: Convert to Exponential Form.

Sol'n $2.3 = \log\left(\frac{x}{1.9}\right)$

$10^{2.3} = \frac{x}{1.9}$, convert to exponential aka def'n of log

$x = 379.1$, calculator.

Use your knowledge of:

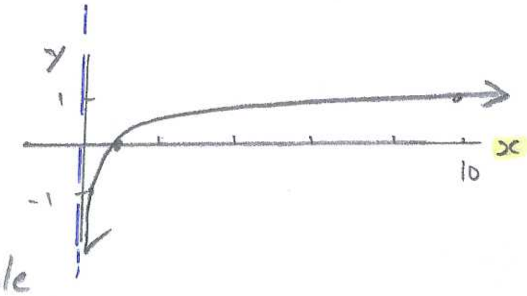
1) Solving equations
2) Laws of Logs
3) $a^y = x \iff \log_a x = y$, def'n of log.
4) $a^{\log_a x} = x$, inverse property
5) $\log_a 1 = 0$, property of logs.

b) $\log_3 x = 5$

Sol'n $3^5 = x$, by def'n.

$x = 243$

Ex₂ Solve $\log_2(x+1) - \log_2(x-1) = \log_2 4$



Sol'n (Recall): Domain of log function is > 0

$\log_2\left(\frac{x+1}{x-1}\right) = \log_2 4$, Quotient Rule

$2^{\log_2\left(\frac{x+1}{x-1}\right)} = 2^{\log_2 4}$, Raised both sides to exponent position. aka "Power it."

(e)

$$\cancel{2}^{\log_2\left(\frac{x+1}{x-1}\right)} = \cancel{2}^{\log_2 4}$$

$$\frac{x+1}{x-1} = 4 \text{ , inverse property}$$

$$x+1 = 4(x-1)$$
$$x+1 = 4x-4$$
$$5 = 3x$$
$$\frac{5}{3} = x$$

Restrictions: Domain > 0

So, $x+1>0$ and $x-1>0$

So, $\underbrace{x>-1 \quad , \quad x>1}$

Together, $x>1$

So $x=\frac{5}{3}$ is admissible ✓

Ex_3 Solve a) $2\log x = \log 8 + \log 2$

Sol'n

Power Law $\rightarrow$ $\log x^2 = \log 16$ $\leftarrow$ Product Law

$$10^{\log x^2} = 10^{\log 16} \text{ , power it}$$
$$x^2 = 16 \text{ , delogged}$$
$$x = \pm 4$$

But domain of log function >0

So $x>0$. Thus $x=-4$ inadmissible, and $x=4$ ✓

b) $\log_8(2-x)+\log_8(4-x)=1$

Sol'n $\log_8(2-x)(4-x) = \log_8 8$

Built Like Logs ✓

$\therefore (2-x)(4-x)=8$, "delog"

$$x^2-6x+8=8$$
$$x^2-6x=0$$
$$x(x-6)=0$$

reject

$\therefore x=0$ or $\cancel{x=6}$

Restrictions: $2-x>0$ and $4-x>0$

So $x=0$ ✓

Note$_1$: Ex_3 a) and Ex_3 b) solutions had quadratic log equation lines.

Note$_2$: You can always verify your answer using a LS-RS check.

Ex_3 a) Check: $LS = 2\log(4) \doteq 1.2041$ $\quad RS = \log 8 + \log 2 \doteq 1.2041$ $\quad LS \doteq RS$ ✓

Ex_4 Solve $6^{3x} = 4^{2x-3}$ $\leftarrow$ An exponential equation.

Sol'n Using yesterday's strategy #2.

$$\log 6^{3x} = \log 4^{2x-3}$$
$$3x\log 6 = (2x-3)\log 4$$
$$3x\log 6 = 2x\log 4 - 3\log 4$$
$$3x\log 6 - 2x\log 4 = -3\log 4$$
$$\frac{x(3\log 6 - 2\log 4)}{(3\log 6 - 2\log 4)} = \frac{-3\log 4}{(3\log 6 - 2\log 4)}$$

$\rightarrow x \doteq -1.598$

$_N$P147 # 7-9, 14, 17-20

[$_A$P493 # 4, 11, 12a-c]