2.6 * Growth and Decay Story Questions and Logs *

Warmup 1. Solve $4^x = 8^{x+3}$

Soln: $(2^2)^x = (2^3)^{x+3}$, build like bases

$2^{2x} = 2^{3x+9}$, equal powers

$\therefore 2x = 3x+9$, set exponents equal

$\therefore -9 = x$

Soln 2: $\log 4^x = \log 8^{x+3}$

$(x)\log 4 = (x+3)\log 8$, power law

$\frac{x}{x+3} = \frac{\log 8}{\log 4}$ ← change of base formula

$\frac{x}{x+3} = \log_4 8$,

$\frac{x}{x+3} = \frac{3}{2}$, evaluate the log.

$2x = 3x+9$, cross-multiply

$-9 = x$

2. 12g of tritium decay to 9.25g in 2.5 years. Estimate the half-life.

Soln: $\therefore y = c(a)^x$ ← # decay periods

$\therefore 9.25 = 12\left(\frac{1}{2}\right)^{\frac{2.5}{h}}$ ← h is half life duration

"We know $h \neq 2.5$ years, Otherwise future value would be 6 grams in this story. Thus $h > 2.5$ years"

$0.7708\overline{3} = \left(\frac{1}{2}\right)^{\frac{2.5}{h}}$

$\log 0.7708\overline{3} = \log\left(\frac{1}{2}\right)^{\frac{2.5}{h}}$

$\log 0.7708\overline{3} = \frac{2.5}{h}\log 0.5$, drop down

$\frac{\log 0.7708\overline{3}}{\log 0.5} = \frac{2.5}{h}$

$0.3755 = \frac{2.5}{h}$

→ $h \doteq 6.66$ years.

So half life about 6.66 years

Check: LS = 9.25 | RS = $12\left(\frac{1}{2}\right)^{\frac{2.5}{6.66}}$ = 9.251 (Close enuff :))

Ex, Caffeine has a half life of about 5.5h. If Ali ingested 62.5 mg, when will the amount remaining in the blood be 4mg?

Soln "Helpful Chart"

t	Mass (g)
0	62.5
5.5	31.25
11	15.625
16.5	7.81
22	3.91

Close to 22h > 22; 3.91 < Close to 4g

Half life loses 50% every period

$y = 4$

$c = 62.5$

$a = \frac{1}{2}$.

$x = \frac{t}{5.5}$ ← half-life, h

$y = c(a)^x$ ← # of decay periods

$4 = 62.5\left(\frac{1}{2}\right)^{\frac{t}{5.5}}$ ← Always same units

$0.064 = \left(\frac{1}{2}\right)^{\frac{t}{5.5}}$

$\log 0.064 = \frac{t}{5.5}\log\left(\frac{1}{2}\right)$

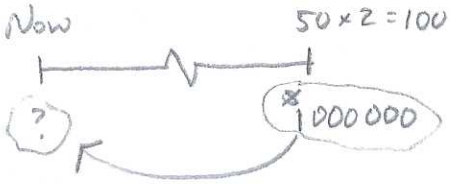
$t = \frac{5.5\log 0.064}{\log 0.5}$

$t = 21.81$ hours

So after about 21.8 days, drug decayed to less than 4mg.

Ex 2 Calculate the present value of $1,000,000 which was invested for 50 years at 4.5%/a compounded semi-annually.

Soln From 3U, helpful pic:



"Divide for i
Multiply for n"

$A = P(1+i)^n$, from 3U compound interest formula

A = future value
P = present value
a = 1 + i ← interest rate per growth period
n = # of growth periods
$i = \frac{0.045}{2}$ ← For semi-annual

$1000\,000 = P\left(1 + \frac{0.045}{2}\right)^{50\times 2}$, sub in

$1\,000\,000 = P(9.2540)$, evaluated power

$\therefore P = \$108,061.38$, divided

So no need to have $1 million now to have $1 million later.
The power of compounded interest grows your money.

Ex 3 The battery of a cell phone loses 30% of its charge each day. Assuming battery is 100% charged, when will charge remaining be 15%?

Soln

$y = c(a)^x$ for decreasing battery charge

$15 = 100(0.70)^x$

y = 15
c = 100
a = 1 − 0.30 = 0.70
x = ?

$0.15 = 0.70^x$, powers equal

$\log 0.15 = x \log 0.70$, log it and power law

$\frac{\log 0.15}{\log 0.70} = x$

$x \doteq 5.32$ days

So about 5.32 days later.

N P132 # 7-12, 20

N P140 # 6, 7, ≠ (9, 10) every other letter, 11

Look up formulas and examples on pgs 138, 139.