

1. If $0 \leq \theta \leq 2\pi$, find the possible measure of θ .

(a) $\tan \theta = -\sqrt{3}$

(b) $\cos \theta = -\frac{1}{2}$

(c) $\sin \theta = 0$

(d) $\cos \theta = \frac{\sqrt{3}}{2}$

2. For each function, state the amplitude, period, phase shift, vertical translation, and sketch them.

(a) $y = 2 \sin (2x - \pi) + 1$

Amplitude _____ Period _____ Phase shift _____ Vertical translation _____

(b) $y = -2 \cos 2(x - \pi) - 2$

Amplitude _____ Period _____ Phase shift _____ Vertical translation _____

3. Factor fully:

(a) $24x^2 - 50x - 14$

(b) $25a^2 - 81b^2$

(c) $4x^2 + y^2 - 1 + 4xy$

4. (a) Find the family of cubic functions whose x -intercepts are $-1, 3, 3$.

(b) Find the particular member of this family whose graph passes through $(-2, 40)$.

5. Determine the remainder, using the Remainder Theorem, in the division:

$$(5x^3 + 7x^2 - 3x + 11) \div (3x + 2)$$

6. Solve for x . Show your work and include i in the final answer. ($x \in C$, the complex numbers.)

(a) $2x^2 - 5x + 5 = 0$

(b) $24x^3 + 81 = 0$

7. A construction worker drops a bolt while working on a high-rise building 220 m above the ground. After t seconds, the bolt has fallen a distance of s metres, where $s(t) = 220 - 5t^2$, $0 \leq t \leq 8$.

(a) Find the average rate of change during the third second.

(b) Find the average rate of change for the interval $3 \leq t \leq 8$.

(c) Find the rate of change at $t = 2$.

8. If $f(x) = x^3 - 9x^2 + 22x - 8$,

(a) What is the linear factor of $f(x)$?

(b) Find the quadratic factor of $f(x)$.

9. Find the value of k if there is a remainder of -21 when $x^3 + 2x^2 + kx - 3$ is divided by $x + 1$.

10. Find the exact number of degrees in the angles with the following radian measures.

(a) $\frac{\pi}{3}$

(b) $\frac{3}{4}\pi$

(c) 5π

(d) $\frac{11}{3}\pi$

11. Find the exact radian measure in terms of π for each of the following angles.

- (a) 45° (b) 225° (c) 150° (d) 300°

12. Sketch the graph of the following function.

$$y = -2 \cos 2x + \frac{\rho}{3} - 1, -\rho \leq x \leq \rho$$

13. Solve the following equations:

- (a) $2 \cos 2x - \sqrt{3} = 0$ (b) $2 \cos^2 x + 3 \sin x - 3 = 0$
 (c) $\sin^2 x - 2 \sin x - 3 = 0$ (d) $\sin x - 2 \sin x \cos x = 0$

14. Functions $R(x) = -2x^2 + 8x$ and $C(x) = 3x + 2$ are the estimated revenue and cost functions for the manufacture of a new product.

- a) Determine the average profit function $AP(x) = \frac{P(x)}{x}$. Express this function in two different forms.
 b) What are the break-even quantities for the profit function?

15. Find the equation of the horizontal asymptote of each curve.

- (a) $f(x) = \frac{2x - 3}{5 - x}$ (b) $g(x) = \frac{x}{x^2 - 4}$

16. Let $y = \frac{x^2}{x^3 - 2x^2 - 5x + 6}$. Find the domain, intercepts, and vertical and horizontal asymptotes.

Then use the information to sketch an approximate graph. Describe behaviour about the asymptotes.

17. The half-life of the isotope Sodium-21 ${}_{11}\text{Na}^{21}$ is 22.5 seconds. How much of an initial sample of 300 g will be left after 5 minutes?

18. Use squared paper to plot the graph of $y = 2 \log(x - 2) + 2$ labeling: the x-intercept, the vertical asymptote, and two important points.

19. Complete the following chart.

Logarithmic Form	Exponential Form
$\log_5 25 = 2$	
	$3^{-2} = \frac{1}{9}$
$\log_2 32 = 5$	
	$7^0 = 1$

20. Complete the chart.

function	horizontal asymptote	y-intercept	growth or decay
$y = 2^x + 3$			
$y = \left(\frac{1}{2}\right)^x - 2$			

21. For the function $y = 3\left(\frac{1}{2}\right)^x + 2$

- State the equation of the horizontal asymptote.
- State the Y-intercept.
- State the domain and range.
- Sketch the graph on squared paper showing the Y-intercept, the horizontal asymptote, $f(-1)$ and $f(1)$.

22. Simplify each of the following:

(a) $\frac{(7x^{-3}y^0)^3}{(49x^2)^{\frac{1}{2}}}$

(b) $\frac{x^{\frac{5}{2}} - x^{\frac{-1}{2}}}{x^{\frac{-1}{2}}}$

(c) $\frac{(x^2y \div xy^3)^3}{x^3y^2}$

23. The sound level of a moving power lawn is 109 dB. The noise level in front of the amplifiers at a concert is about 118 dB. How many times louder is the noise at the front of the amplifiers than that of a moving power lawn mower?

24. An earthquake scored 2.8 on the Richter scale in city A. At city B, it scored 5.3. How much more intense was the earthquake at city B than the earthquake at city A?

25. Solve for x

(a) $\log_{16} x = \frac{3}{2}$

(b) $\log_x 8 = \frac{1}{2}$

(c) $\log_2(x+3) + \log_2(x-4) = 3$

(d) Use your calculator to solve for x: $17^x = 73$. (Answer to three decimal places)

26. Prove the following identities:

(a) $\frac{\sin x + \tan x}{1 + \cos x} = \tan x$

(b) $\frac{-\sin^2 x}{(\cos x + 1)^2} = \frac{\cos x - 1}{\cos x + 1}$

(c) $\tan x + \frac{1}{\tan x} = \frac{1}{\sin x \cos x}$

(d) $\cot 2\alpha = \frac{\cot^2 \alpha - 1}{2 \cot \alpha}$

27. Sketch on squared paper $y = (x+2)(x-1)(x-3)$ by using x-intercepts and y-intercepts.

28. Neeru is holding a year-end clearance sale in her clothing store. All prices are discounted by 25%.
- Write an equation that expresses the sale price of an item as a function of its original price.
 - If the sale price of a shirt is \$42, what was its original price?

29. Sketch at least two cycles of the graph of each of the following.

- $y = 3 \cos x - 2$
- $y = -2 \cos \left(2x + \frac{\pi}{3} \right) - 1$
- $y = -2 \sin 2x + 2$

30. Solve each of the following for x , $x \in \mathbb{R}$, and graph the solution

- $x(x-3) < 0$
- $(x+3)(x-1) \leq 0$
- $x^2 - 7x + 10 \geq 0$

31. Find the equation of the horizontal asymptote of each curve. Then sketch.

(a) $f(x) = \frac{2x-3}{5-x}$ (b) $f(x) = \frac{x}{x^2-4}$ (c) $f(x) = \frac{x^2+1}{x^2-1}$

32. Find an equation of the oblique asymptote of each curve. Then sketch. Determine the intervals of increase and decrease.

(a) $f(x) = \frac{3x^2 - 4x + 5}{x}$ (b) $h(x) = \frac{x^2 - 4}{x^2}$ (c) $f(x) = \frac{x^3 + 5x^2 + 3x + 10}{x^2 + 2}$

33. Solve.

(a) $2^{x+1} = 2x + 3$ (b) $\sin \pi x = |x|$ (c) $\log x - 1 = x^2 - 3$

Answers:

1. $\frac{2\pi}{3}, \frac{5\pi}{3}; \frac{2\pi}{3}, \frac{4\pi}{3}; 0, \pi, 2\pi; \frac{\pi}{6}, \frac{11\pi}{6}$

2. $2, \pi, \frac{\pi}{2}, 1; 2, \pi, \pi, -2$

3. $2(4x+1)(3x-7); (5a-9b)(5a+9b); (y+2x+1)(y+2x-1)$

4. $y = k(x+1)(x-3)^2; y = \frac{-8}{5}(x+1)(x-3)^2$

5. $14.\overline{629}$

6. $x = \frac{5 \pm \sqrt{-15}}{4}; \left\{ \frac{-3}{2}, \frac{3 \pm 3\sqrt{3}i}{4} \right\}$

7. $-25m/s, -55m/s; -20m/s$

8. $(x-4); x^2 - 5x + 2$

9. 19

10. $60^\circ, 135^\circ, 900^\circ, 660^\circ$

11. $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{3}$

12. Graphing Calculator

13.

$$\frac{\pi}{12} + k\pi, \frac{11\pi}{12} + k\pi, k \in I; \frac{\pi}{2} + 2k\pi, \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi, k \in I; \frac{3\pi}{2} + 2k\pi, k \in I; \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi, \pi k, k \in I$$

14. $-2x^2 + 5x - 2$ aka $-2x + 5 - \frac{2}{x}; 2, 0.5$

15. $y = -2; y = 0$

16. Graphing Application

17. $0.0097\% = 0.029g$

18. Graphing Application

19. $5^2 = 25, \log_3 \frac{1}{9} = -2, 2^5 = 32, \log_7 1 = 0$

20. $y = 3, (0,4), \text{growth}; y = -2, (0,-1), \text{decay}$

21. $y = 2; (0,5); D = \{x \in R\}, R = \{y \geq 2, y \in R\};$ Graphing App

22. $49x^{-10}, x^3 - 1, y^{-8}$

23. $10^{0.9} = 8$ times

24. $10^{(5.3-2.8)} = 316$ times

25. $x = 64; x = 64; x = 5; x = 1.514$

26. All QED

27. Graphing App

28. $S(p) = 0.75p; \$56$

29. Graphing App

30. $0 < x < 3; [-3,1]; x \leq 2$ or $x \geq 5$

31. H.A.: $y = -2; y = 0; y = 1.$ Graphing App

32. O.A.: $y = 3x - 4; y = x; y = x + 5.$

Graphing App.

$(-\infty, -1.29) \cup (1.29, \infty), (-1.29, 0) \cup (0, 1.29); (-\infty, -1.91) \cup (0, \infty), (-1.91, 0);$ all x

33. Graphing App.