

MHF 4UI Exam Review Worksheet

1. Simplify

a) $6(2x^2 - 5x) - 14x(3 - x)$

b) $\frac{-12x^5y^4 + 2x^2y^3}{2x^2y^3} + x^3y$

2. Factor each of the following completely

a) $(5x - 4)^2 - 3(5x - 4) + 2$

b) $9(x + 2y)^2 - 16(x - 2y)^2$

c) $x^4 - 38x^2z^2 + 72z^4$

d) $27y^3 + 64$

3. Simplify each of the following, and state restrictions

a) $\frac{3x}{x^2 + 3x + 2} - \frac{4x}{x^2 + 5x + 6}$

b) $\frac{x^2 - 16y^2}{x^2 - 2xy} \times \frac{x^2 - 4xy}{x^2 - 6xy + 8y^2} \div \frac{x^2 + 4xy}{x - 2y}$

4. If $f(x) = 3x + 5$ and $g(x) = x^2 + 3x - 5$, determine

a) $f(-3) - g(-2)$

b) $P(x) = g(f(x))$

5. Given the function $f(x) = \begin{cases} -x - 1 & \text{for } -4 \leq x \leq -1 \\ x + 1 & \text{for } -1 \leq x \leq 0 \\ -x + 1 & \text{for } 0 \leq x \leq 4 \end{cases}$

Plot the function $f(x)$ on a grid.6. Given $f(x) = \frac{1}{x}$, the following transformations are performed in the order given below

1. Reflection in the x - axis.
2. Horizontal compression by a factor of 4.
3. Horizontal translation by 1 unit to the left.
4. Vertical translation by 3 units up.

a) Find the *algebraic form* of the resulting function

7. Solve the equation $\frac{3}{x+1} - \frac{2}{x-1} + \frac{4x}{x^2-1} - 3 = 0$

8. State the domain and range of each of the following functions

a) $y = -3(x - 4)^2 + 9$

b) $g(x) = \frac{1}{x+6} + 5$

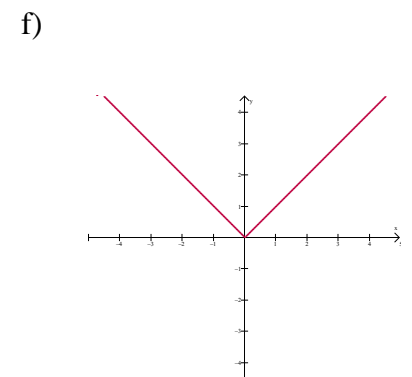
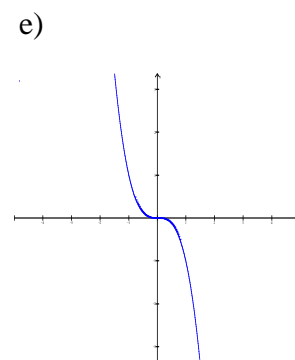
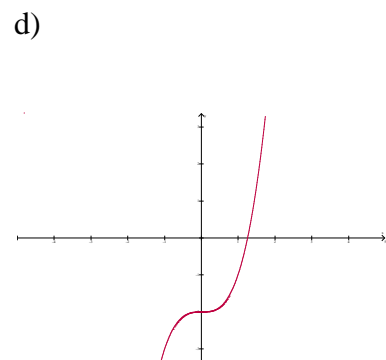
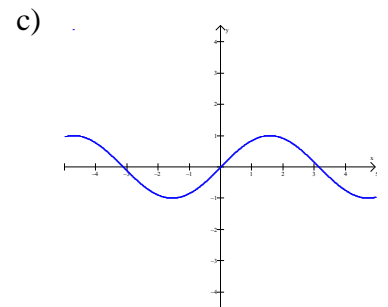
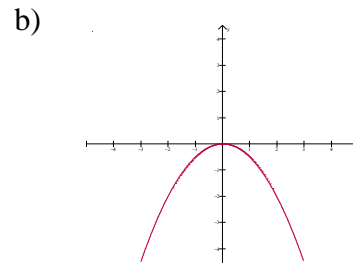
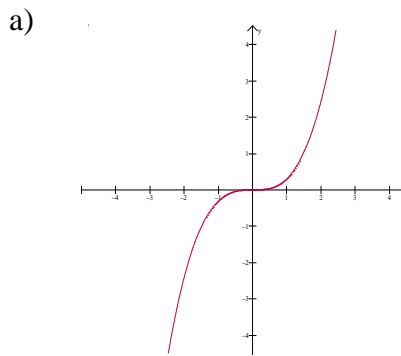
c) $h(x) = -\sqrt{2x+6}$

9. a) Determine the algebraic form of the inverse of $f(x) = \sqrt{\frac{9-x}{3}} + 4$, and state the domain and range of the inverse.

10. Solve and graph the solution of the following inequality on a number line

$$3(2-x)-1 \geq 1-2(3-x)$$

11. Determine which of the following are even, odd or neither.

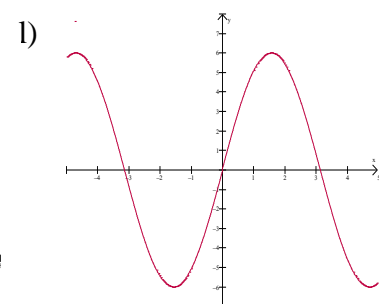
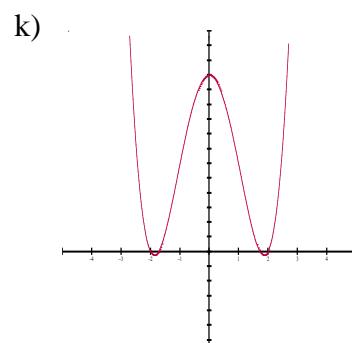


g) $f(x) = 2x^2 + |x|$

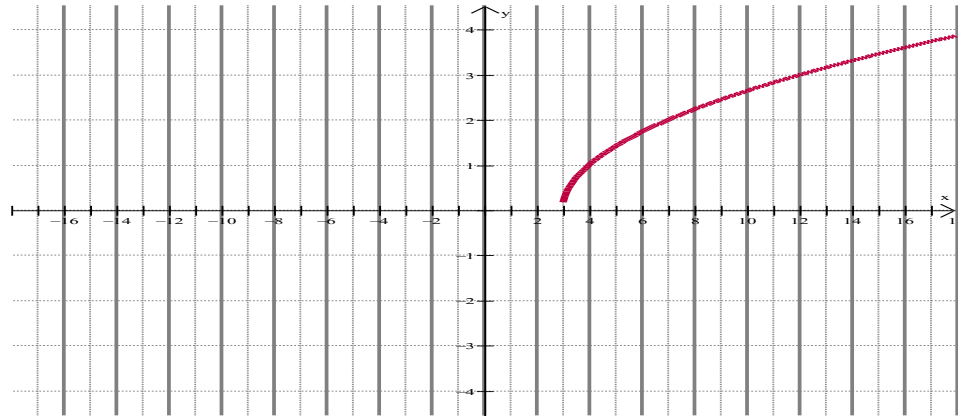
h) $f(x) = x^3 - 2x^2 + x$

i) $y = \frac{x}{2-x}$

j) $f(x) = \frac{1+x^2}{x^3}$



12. Graph shown below represents *half of the function* $f(x)$. Construct the **full graph** if $f(x)$ is an *odd function*. State the **Domain** of the function.



13. Show that

a) $f(x) = \frac{3^x + 3^{-x}}{3x^5 - 27x^3 + x}$ is an odd function b) $g(x) = \frac{x^3 - x}{x^5}$ is an even function

14. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 16$ find the following if it exists.

a) $(f \circ g)(x)$

b) $(g \circ f)(x)$

- c) State the domain and range using interval notation for your answers above

15. Given $f(x) = 15 - x^2$ and $g(x) = \sqrt{x+1}$ find the **domain** and **range** of

a) $f \circ g$

b) $g \circ f$

16. Given $f(x) = \sqrt{x-5}$ and $g(x) = x^2 + 6$ find the following if it exists.

a) $(g \circ f)(x)$

b) $f^{-1}(x)$

c) $(f^{-1} \circ f)(x)$

- d) State the domain and range for the above .

17. Divide using a method of your choice.

a) $\frac{3y^3 - 2y^2 + 12y - 9}{y^2 + 2}$

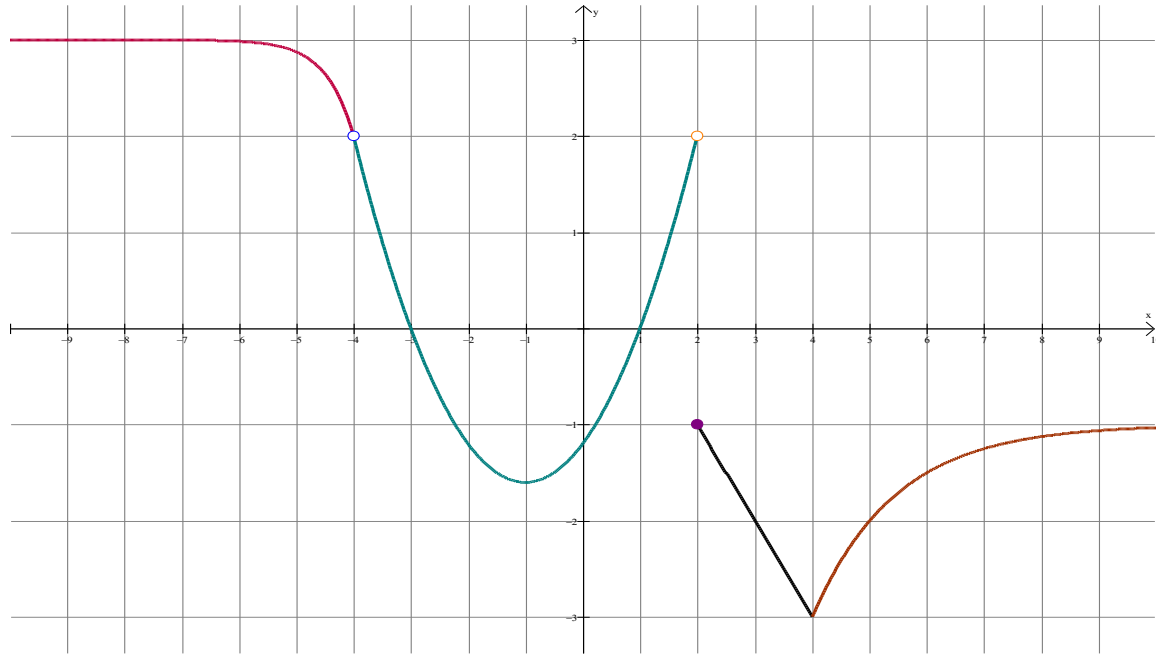
b) $\frac{x^3 - 2x^2 - 5x + 6}{x - 1}$

18. Determine the quotient and the remainder of the division of

$P(x) = -3x^6 + 15x^4 - 3x^2 - 2x + 1$ by $d(x) = x^2 - 4x + 5$

19. The divisions $\frac{2x^3 + 4x^2 - Ax + 5}{x + 3}$ and $\frac{6x^3 + 4x^2 - Ax + 5}{x - 1}$ have the same remainder. Find the value of A .
20. When $x^3 + kx^2 - 4x + 2$ is divided by $x + 2$ the remainder is 26, find k .
21. When $rx^3 + gx^2 + 4x + 1$ is divided by $x - 1$ the remainder is 12 and when divided by $x + 3$ the remainder is -20. Find r and g .
22. Factor the following.
- a) $5x^3 - 7x^2 - x + 3$ b) $2x^4 - 3x^3 + 3x^2 + 3x - 5$
23. Find the roots for the following.
- a) $-5(x - 3)(2x + 7)(x + 1) = 0$ b) $x^4 + 2x^3 - 4x^2 - 8x = 0$
 c) $12x^3 + 20x^2 + x - 3 = 0$
24. Sketch the following. Show your work. Label all intercepts.
- a) $y = -3(x - 2)^2(x + 5)(x - 8)$ b) $y = x^3 - 4x^2 - 2x + 20$
25. Sketch a possible graph of a function which satisfies each set of conditions.
- a) Degree 4 , negative leading coefficient, 2 distinct roots, 3 turning points b) Degree 5 , positive leading coefficient and 3 distinct real roots
26. Determine the zeroes and sketch each of the following functions
- a) $f(x) = -(x + 4)^5(x - 3)^2(x - 6)(x - 8)$
 b) $M(x) = 4x^4 - 4x^3 - 9x^2 + x + 2$
27. Find the value(s) of m for which $(m - 1)x^2 + mx + 1 = 0$ has no real roots.
28. Solve graphically and algebraically
- a) $-8x^2 + 2x + 1 \geq 0$ b) $(2x - 3)(x + 4)(x + 5)(x - 2) \leq 0$ c) $\frac{(x + 7)(x - 2)}{(x - 6)(x + 5)} \leq 0$
 d) $x^3 - 2x^2 - 9x + 18 \leq 0$ e) $x^4 - x^3 - 11x^2 + 9x > -18$

29. Use the graph of $f(x)$ below to determine each of the following



a) $\lim_{x \rightarrow -\infty} f(x) - \lim_{x \rightarrow +\infty} f(x) = \underline{\hspace{2cm}}$ b) $\lim_{x \rightarrow -4} f(x) = \underline{\hspace{2cm}}$ c) $\lim_{x \rightarrow 2^-} f(x) - \lim_{x \rightarrow 2^+} f(x) = \underline{\hspace{2cm}}$

30. Consider the graph and determine the limits:

a) $\lim_{x \rightarrow -2} f(x) = \underline{\hspace{2cm}}$

b) $\lim_{x \rightarrow 2^-} f(x) = \underline{\hspace{2cm}}$

c) $\lim_{x \rightarrow 2^+} f(x) = \underline{\hspace{2cm}}$

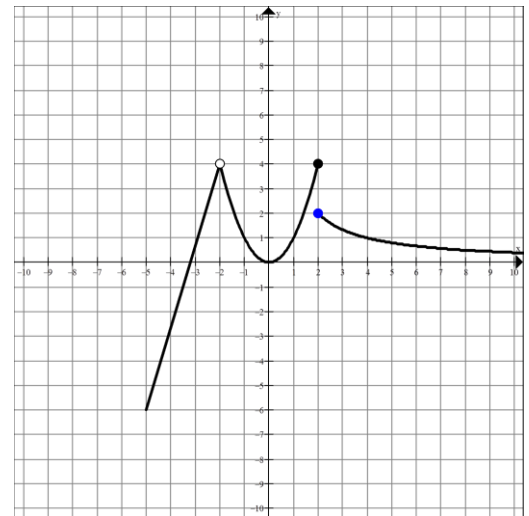
d) $\lim_{x \rightarrow 2} f(x) = \underline{\hspace{2cm}}$

e) $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$

f) $\lim_{x \rightarrow 0} f(x) = \underline{\hspace{2cm}}$

g) $\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$

$y = f(x)$



31. Sketch the following piecewise function.

$$g(x) = \left\{ \begin{array}{ll} (x+3)^2 - 4 & \text{if } x \in (-\infty, -1) \\ x^2 + 3 & \text{if } x \in [-1, 1) \\ 3x + 1 & \text{if } x \in [1, \infty) \end{array} \right\}$$

Find the limits.

a) $\lim_{x \rightarrow 1} g(x) = \underline{\hspace{2cm}}$

b) $\lim_{x \rightarrow -1} g(x) = \underline{\hspace{2cm}}$

c) $\lim_{x \rightarrow -5} g(x) = \underline{\hspace{2cm}}$

Identify all discontinuities and state the type of discontinuity.

36. Given $W(x) = \frac{7 \times 5^x - 12 \times 5^{-x}}{3 \times 5^x + 8 \times 5^{-x}}$ evaluate $\lim_{x \rightarrow +\infty} W(x) - \lim_{x \rightarrow -\infty} W(x)$

37. Given $f(x) = x^4$ determine $R(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

38. If $L(x) = \frac{x}{3x-1}$, determine $\lim_{h \rightarrow 0} \frac{L(x+h) - L(x)}{h}$

39. Find the equation of a tangent line to $y = x^3$ at $x = -2$.

40. Find the equation of the line **perpendicular** to the graph of $f(x) = 3x^2 + 13x - 3$ at the point $P(1, 13)$. Please use *first principles/limits*.

41. The position of an object from a reference point is modeled by the equation $D(t) = 4t^2 - 3t + 5$. Here (t) is the time measured in seconds and (D) is the distance measured in meters.

a) Determine the average velocity of the object over the time interval $t \in [3, 7]$ seconds.

b) Determine the instantaneous velocity of the object at $t = \frac{3}{8}$ seconds using *first principles/limits*.

42. The displacement, in metres, of a particle moving in a straight line is given by $s = t^2 - 4t + 3$ where t is measured in seconds.

a. Find the average velocity for $3 \leq t \leq 5$.

b. Find the instantaneous velocity when $t = 4$.

43. If the Surface Area of a sphere is $A = 4\pi r^2$, find the rate of change of surface area with respect to radius when $r = 7$.

44. Find an expression for the instantaneous rate of change of each of the following.

a) $f(x) = \frac{1}{3x^2 - 2}$

b) $y = \sqrt{2x+5}$

Answers

1a) $26x^2 - 72x$ b) $-5x^3y + 1$

2a) $5(x-1)(5x-6)$ b) $-(x-14y)(7x-2y)$ c) $(x-6z)(x+6z)(x^2-2z^2)$

d) $(3y+4)(9y^2-12y+16)$

3a) $\frac{-x(x-5)}{(x+1)(x+3)(x+2)}, x \neq -1, -2, -3$ b) $\frac{x-4y}{x(x-2y)}, x \neq 0, 2y, \pm 4y$

4a) 3 b) $9x^2 + 39x + 35$

5) Graphing App

6) $y = -\left(\frac{1}{4(x+1)}\right) + 3$

7) $x = \frac{2}{3}$

8a) $x \in R, y \leq 9$ b) $x \neq -6, g(x) \neq 5$ c) $x \geq -3, h(x) \leq 0$

9) $f^{-1}(x) = -3(x-4)^2 + 9, x \geq 4, f^{-1}(x) \leq 9$

10) $x \leq 2$

11) odd, even, odd, neither, odd, even, even. neither, neither, odd, even, odd

12) $\{x \in R, x \geq 3 \text{ or } x \leq -3\}$ Graphing App

13a) odd b) even

14a) $\sqrt{x^2 - 16}$ b) $x - 16$ c) i) $D = (-\infty, -4] \text{ or } [4, \infty)$
 $R = [0, \infty)$ ii) $D = [0, \infty)$
 $R = [-16, \infty)$

15a) $(f \circ g)(x) = -x + 14, x \geq -1, (f \circ g)(x) \leq 15$

b) $(g \circ f)(x) = \sqrt{16 - x^2}, -4 \leq x \leq 4, 0 \leq (g \circ f)(x) \leq 4$

16a) $(g \circ f)(x) = x + 1$ b) $f^{-1}(x) = x^2 + 5$ c) $(f^{-1} \circ f)(x) = x$

17a) $3y - 2 + \frac{6y - 5}{y^2 + 2}$ b) $x^2 - x - 6$

18) quotient $-3x^4 - 12x^3 - 18x^2 - 12x + 39$ remainder $214x - 194$

19) $A = 7$

20) $K = 6$

21) $r = 2, g = 5$

22a) $(x-1)^2(5x+3)$ b) $(x-1)(x+1)(2x^2-3x+5)$

23a) $x = -1, -\frac{7}{2}, \text{ or } 3$ b) $x = 0, \pm 2$ c) $x = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{3}$

24) Graphing App

25) Graphing App

26) Graphing App

27) no solution

28a) $-\frac{1}{4} \leq x \leq \frac{1}{2}$ b) $-5 \leq x \leq -4 \text{ or } \frac{3}{2} \leq x \leq 2$ c) $-7 \leq x < -5 \text{ or } 2 \leq x < 6$

d) $x \leq -3 \text{ or } 2 \leq x \leq 3$ e) $x < -3 \text{ or } -1 < x < 2 \text{ or } x > 3$

39a) 4 b) 2 c) 3

30a) 4 b) 4 c) 2 d) DNE e) 0 f) 0 g) DNE.

31a) 4 b) DNE c) 0

36) $\frac{23}{6}$ 37) $4x^3$ 38) $-\frac{1}{(3x-1)^2}$

39) $y = 12x + 16$ 40) $y = -\frac{1}{19}x + \frac{248}{19}$

41a) $37m/s$ b) $0m/s$ 42a) $4m/s$ b) $4m/s$

43) 56π 44a) $-\frac{6x}{(3x^2-2)^2}$ b) $\frac{\sqrt{2x+5}}{2x+5}$