7.6 *Radian Measure – Special Angles*

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Warmup 1. Recall… $\pi = 180°$; $1° = \frac{\pi}{180}$; $1\text{ rad} = \frac{180°}{\pi}$;

$\theta = \frac{a}{r}$; Radians have no units!; Multiply by $\frac{180°}{\pi}$ or $\frac{\pi}{180}$.

2. Convert to radians   3. Convert to degrees.

a) $110°$

Sol'n $= \frac{110°\pi}{180}$

$= \frac{11}{18}\pi$

b) $247°$

Sol'n $= \frac{247\pi}{180}$

$\doteq 4.31^r$

a) $\frac{3\pi}{2}$

Sol'n $= \frac{180°}{\pi} \times \frac{3\pi}{2}$

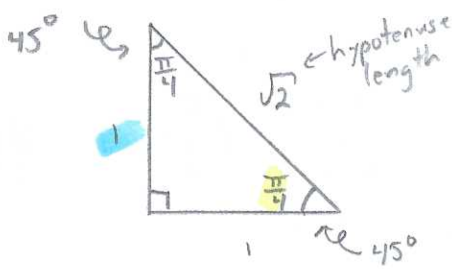
$= 270°$

b) 1.64^r

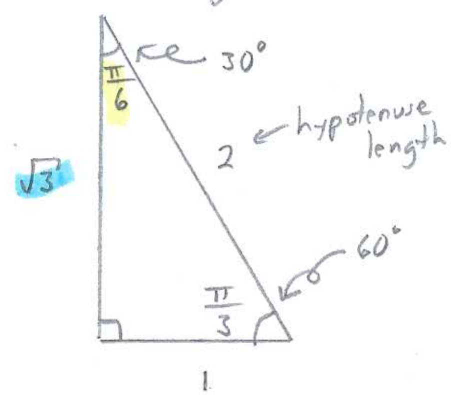
Sol'n $= \frac{180°}{\pi} \times 1.64$

$\doteq 93.97°$

Note₁: We can put the "special isosceles $45°$-$45°$-$90°$ triangle" and the "$30°$-$60°$-$90°$ triangle" into radians.



and



These acute $30° = \frac{\pi}{6}$, $45° = \frac{\pi}{4}$, $60° = \frac{\pi}{3}$ angles occur frequently in trig stories and applications. Thus we should be able to find ratios exactly.

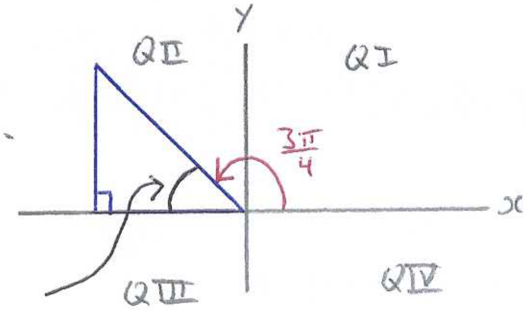
Note₂:

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ ← opposite / ← hypotenuse $\sin\frac{\pi}{6} = \frac{1}{2}$ and $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ ← adjacent / ← hypotenuse $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{\pi}{4} = \frac{1}{1}$ ← opposite / ← adjacent $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ $\tan\frac{\pi}{3} = \frac{\sqrt{3}}{1}$

Note₃: From yesterday, $\cos\frac{3\pi}{4} = -0.7071$. $\cos\frac{3\pi}{4}$ is negative since $\frac{3\pi}{4}$ terminates in Quadrant II. This result infers the CAST rule applies in radians as well! Further, since $\frac{3\pi}{4} = 135°$ is a special angle in Quad II, therefore we can draw and calculate $\cos\frac{3\pi}{4}$ exactly! ↪



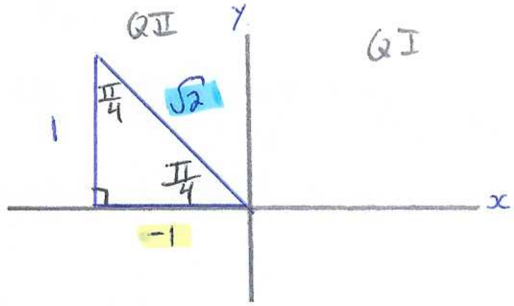
$\frac{3\pi}{4}$ rotation stops at terminal arm

So $\cos\frac{3\pi}{4} = \cos RAA$ in Quadrant II.

Trigonometry means Triangle Measure
Let's build triangles!

Related Acute Angle

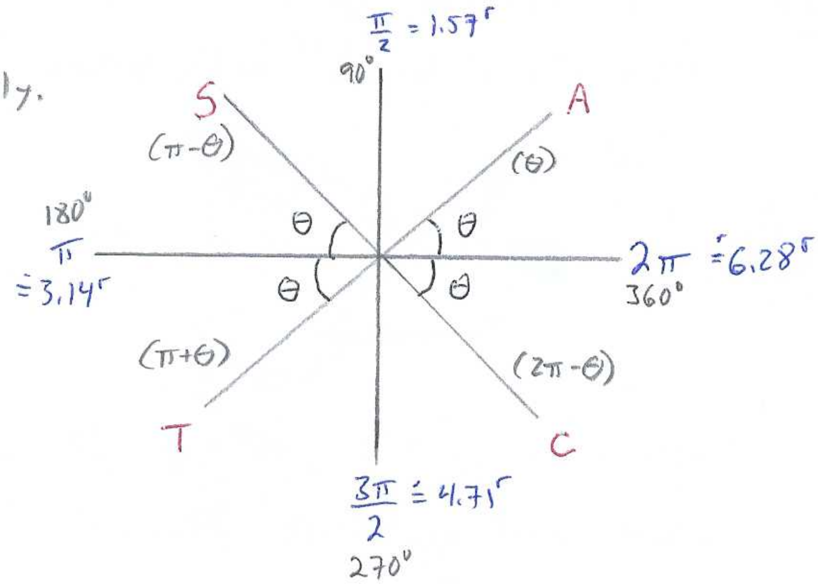
$RAA = \frac{\pi}{4}$ $(180^\circ - 135^\circ = 45^\circ)$



Thus, $\cos\frac{3\pi}{4} = \frac{\text{adjacent length}}{\text{hypotenuse length}}$

$= \frac{-1}{\sqrt{2}}$, exact ✓

$\doteq -0.7071$, approx ✓



Ex₂ Evaluate $\tan\frac{5\pi}{6}$ exactly.

Soln Recall from 3U

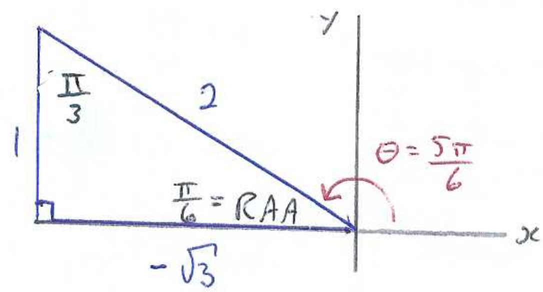
$\tan\frac{5\pi}{6}$

So, $\pi - \phi = \frac{5\pi}{6}$

$\phi = \pi - \frac{5\pi}{6}$

$\therefore RAA = \frac{\pi}{6}$

✓



Thus, $\tan\frac{5\pi}{6} = \frac{1}{-\sqrt{3}}$

$= \frac{1}{-\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{-\sqrt{3}}{3}$, rationalized

✓

"Check using calculator in radian mode"

Def'n, From 3U, the reciprocal trig ratios are

$$\text{cosecant}\,\theta = \csc\theta = \frac{1}{\sin\theta},$$

$$\text{secant}\,\theta = \sec\theta = \frac{1}{\cos\theta},$$

$$\text{cotangent}\,\theta = \cot\theta = \frac{1}{\tan\theta}.$$

Ex3 Evaluate to 2 decimal places.

a) $\csc 80°$

Sol'n $= \dfrac{1}{\sin 80°}$, by def'n

$\doteq 1.02$ in degree mode on calculator.

b) $\cot \frac{\pi}{5}$

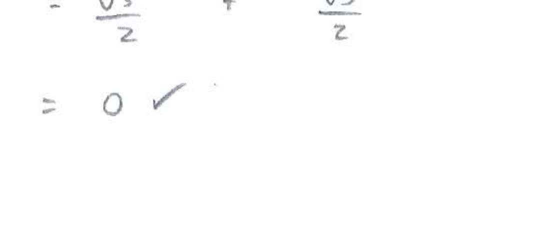
Sol'n $= \dfrac{1}{\tan\frac{\pi}{5}}$, by def'n.

$\doteq 1.38$ in radian mode on calculator

Ex4 Evaluate exactly.

a) $\sin\frac{\pi}{3} + \cos\frac{5\pi}{6}$

Sol'n Draw reference triangles.

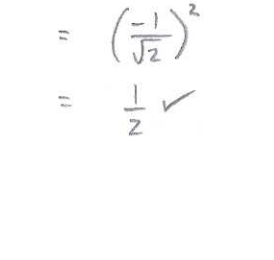


$= \dfrac{\sqrt{3}}{2} + \dfrac{-\sqrt{3}}{2}$

$= 0$ ✓

b) $\cos^2\frac{5\pi}{4}$

Sol'n $= \left(\cos\frac{5\pi}{4}\right)^2$

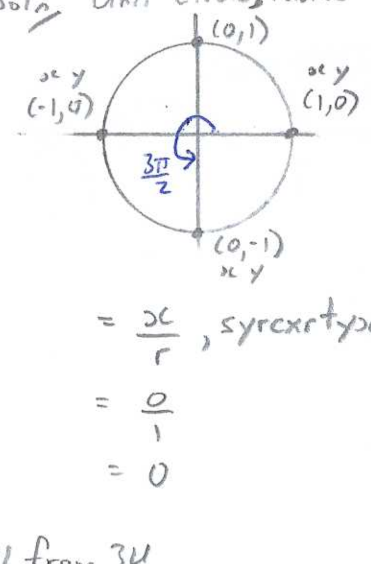


$= \left(\dfrac{-1}{\sqrt{2}}\right)^2$

$= \dfrac{1}{2}$ ✓

c) $\cos\frac{3\pi}{2}$

Sol'n Unit Circle, radius = 1.



$= \dfrac{x}{r}$, syrcxrtyx

$= \dfrac{0}{1}$

$= 0$

Recall from 3U,

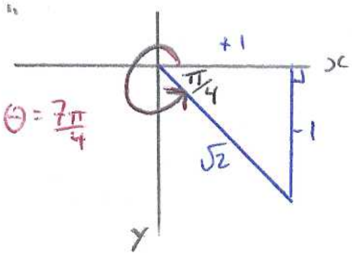
sohcahtoa

= syrcxrtyx on grid.

d) $\csc \frac{7\pi}{4}$

Sol'n $= \dfrac{1}{\sin \frac{7\pi}{4}}$

"RAA $= \frac{\pi}{4}$"



$= \dfrac{1}{\frac{-1}{\sqrt{2}}}$

$= 1 \times \dfrac{\sqrt{2}}{-1}$, copy flip multiply

$= -\sqrt{2}$ ✓

P 278 #1 every other letter

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P 283 #3 every other letter

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Gizmo "Cosine Function"