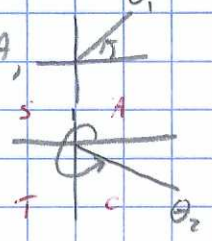
8.8 Dilatations of Trig Functions

Warmup Solve i) $\cos\theta = 0.7315$, $0 \le \theta \le 2\pi$

ii) $\sin\theta = -0.8135$, $0 \le \theta \le 2\pi$.

Soln i) $\theta_1 = \cos^{-1}(0.7315)$

$\doteq 0.7503$, RAA,



"cosine ratio is positive in the C window and A window"

or $\theta_2 = 2\pi - RAA$

$= 2\pi - 0.7503$

$= 5.53^r$

$\therefore$ Roots are aprx 0.75^r and 5.53^r

ii) $\theta = \sin^{-1}(-0.8135)$

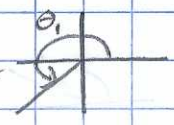
$\doteq -0.9501$

$\therefore RAA \doteq 0.95^r$

$\therefore \theta_1 = \pi - RAA$

$= \pi + 0.95$

$= 4.09^r$ ✓

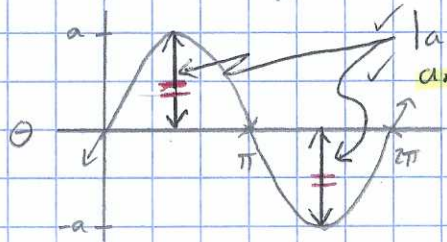


or $\theta_2 = 2\pi - RAA$

$= 2\pi - 0.95$

$= 5.33^r$ ✓

Remark$_1$: In general, $y = a\sin x$ is a vertical dilatation of the basic graph $y = \sin x$. We say $y = a\sin x$ has an Amplitude of a and has been graphed by vertically stretching the graph of $y = \sin x$ by factor a.



✓ $|a|$ = amplitude

✓ amplitude $= \frac{1}{2}$(Max Value − Min Value)

Remark$_2$: The amplitude is half the distance between the maximum and minimum values of the function. It is also the distance from the sinusoidal axis "center line" to the max and min points.

Remark$_3$: In general, $y = \sin kx$ is a horizontal dilatation of the parent curve $y = \sin x$. This horizontal stretch by factor $\frac{1}{k}$ actually changes the period of the function.

Recall, the period of a sine or cosine function is how soon the function repeats itself. The period for the graph of $y = \sin kx$ is $\frac{360^\circ}{|k|}$ aka $\frac{2\pi}{|k|}$ ← "4u"

"3u" →

Ex$_1$ Complete the Table:

Function	Amplitude	k-value	Period
$y = \sin x$	1	1	2π
$y = \sin 2x$	1	2	$2\pi/2 = \pi$
$y = -\sin\frac{1}{3}x$	$\lvert-1\rvert = 1$	$\frac{1}{3}$	$\frac{2\pi}{(\frac{1}{3})} = 6\pi$
$y = 2\cos 3x$	2	3	$\frac{2\pi}{3}$
$y = -\frac{1}{2}\cos\frac{1}{4}x$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{2\pi}{(\frac{1}{4})} = 8\pi$

PART B: Horizontal and Vertical Dilatations

Sketch the graph of each of the following:

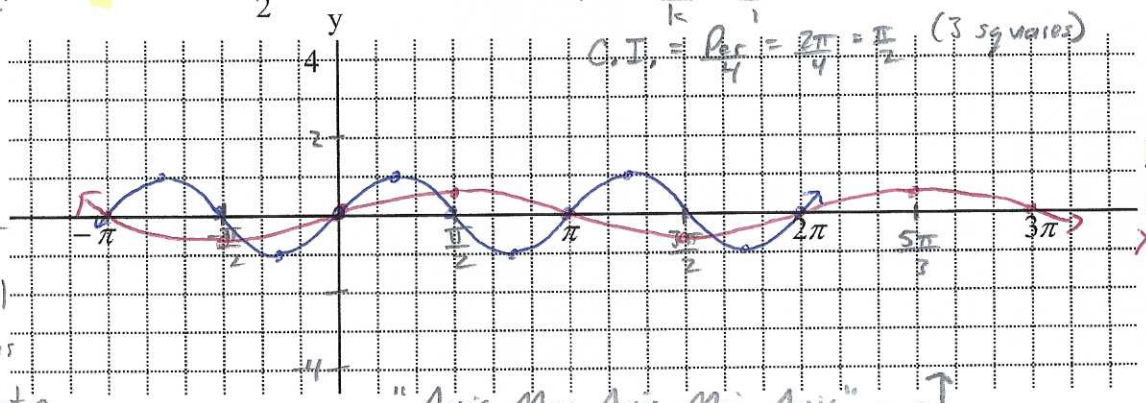
(a) $y = \sin 2x$ and $y = \frac{1}{2}\sin x$

Vertical stretch by factor $\frac{1}{2}$
Amplitude $= \frac{1}{2}$
Per $= \frac{2\pi}{k} = \frac{2\pi}{1} = 2\pi$

Max: $\frac{1}{2}(1) = \frac{1}{2}$.
Min: $\frac{1}{2}(-1) = -\frac{1}{2}$.

Horizontal Stretch by factor $\frac{1}{2}$.
Amp = 1
Period $= \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$
So Critical Interval aka C.I. is $\frac{\pi}{4}$. Thus every 1.5 squares plot a key point.

C.I. $= \frac{\text{Per}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$ (3 squares)

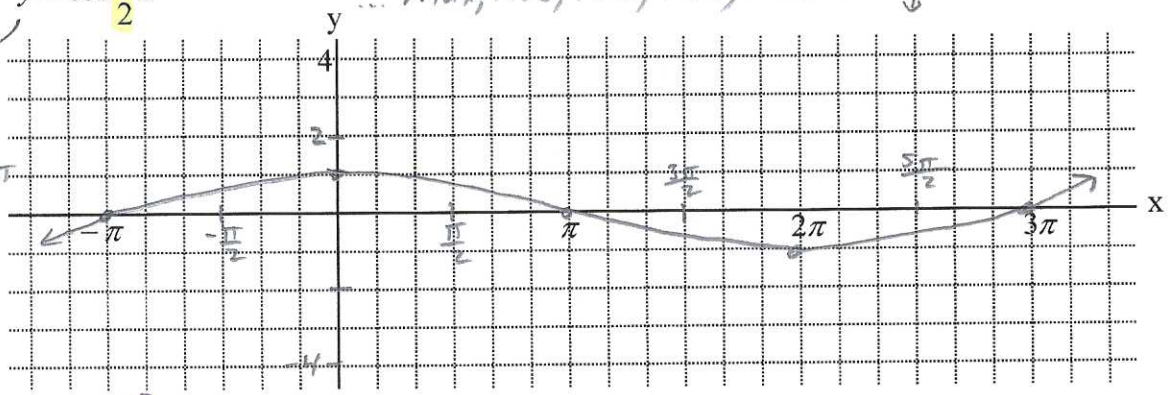


"... Axis, Max, Axis, Min, Axis ..."

"... Max, Axis, Min, Axis, Max ..."

(b) $y = \cos\frac{1}{2}x$

Amp = 1
Per $= \frac{2\pi}{k} = \frac{2\pi}{(\frac{1}{2})} = 4\pi$
C.I. $= \frac{\text{Per}}{4} = \frac{(4\pi)}{4} = \pi$
(6 squares)

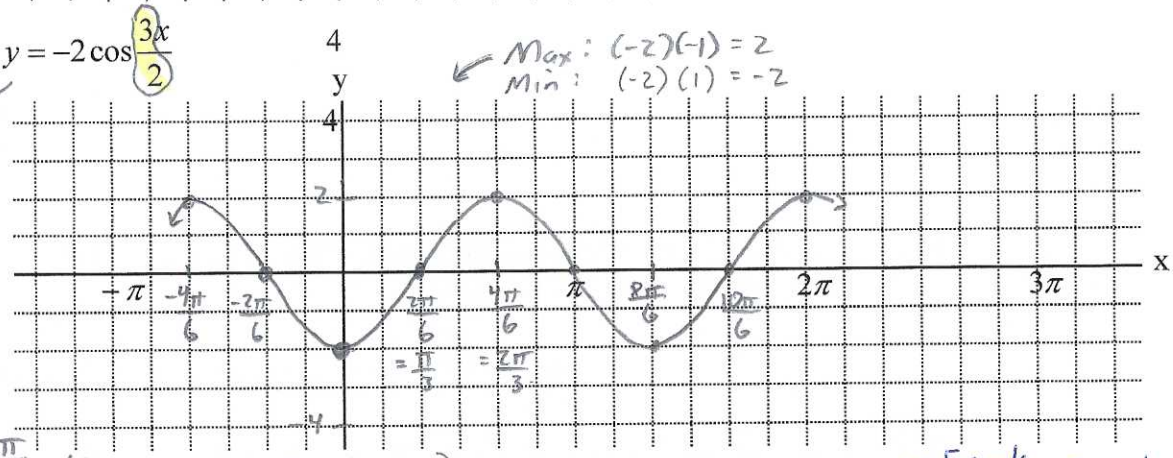


(c) $y = -2\cos\frac{3x}{2}$

Max: $(-2)(-1) = 2$
Min: $(-2)(1) = -2$

Can you see our $k = \frac{3}{2}$?

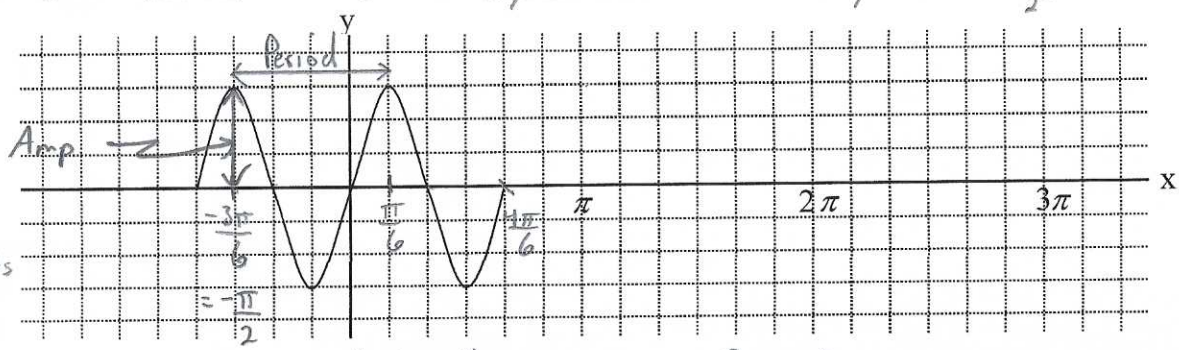
Amp $= |-2| = 2$
Per $= \frac{2\pi}{k} = \frac{2\pi}{(\frac{3}{2})} = \frac{4\pi}{3}$
C.I. $= \frac{\text{Per}}{4} = \frac{(\frac{4\pi}{3})}{4} = \frac{\pi}{3}$ (2 squares since $\frac{\pi}{3} = (2)(\frac{\pi}{6})$)



Write an equation for the following function.

Formula: Period $= \frac{2\pi}{k} = \frac{4\pi}{6} \Rightarrow$ So $k = 3$. (From looking at graph.)

(a) Amplitude: 3 Equation: $y = 3\sin(3x)$ or $y = 3\cos 3(x + \frac{\pi}{2})$ or $y = -3\sin 3(x - \frac{\pi}{3})$ OR ...



"no horizontal nor vertical displacements here" →

Pg 342 # 2, 5ac, 10a ; A P 348 # 2, 3ace.