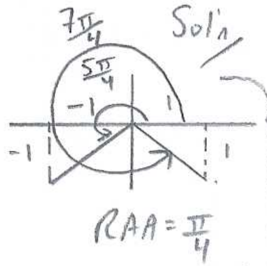
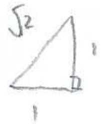
A 8.4 *Quadratic Trig Equations*

Warmup: 1. Evaluate exactly.

a) $2\sin\frac{5\pi}{4}\cos\frac{7\pi}{4}$



RAA $=\frac{\pi}{4}$ (45°)

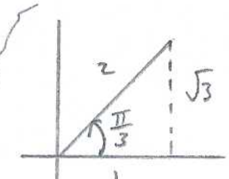


syrcxrtysc

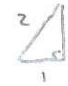
Sol'n
$$= 2\left(\sin\frac{5\pi}{4}\right)\left(\cos\frac{7\pi}{4}\right)$$
$$= 2\left(\frac{-1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{-2}{\sqrt{4}}$$
$$= \frac{-2}{2}$$
$$= -1$$

b) $\cos^2\frac{\pi}{3} - \sin^2\frac{\pi}{3}$

Sol'n
$$= \left(\cos\frac{\pi}{3}\right)^2 - \left(\sin\frac{\pi}{3}\right)^2$$
$$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{1}{4} - \frac{3}{4}$$
$$= \frac{-2}{4} \quad \text{"Fraction Action"}$$
$$= -\frac{1}{2}$$



RAA $=\frac{\pi}{3}$ (60°)



syrcxrtysc

Remark: Usually, but not always, we try to build a product of factors.

Ex. Solve for the exact values of x.

a) $\sin^2 x + 2\sin x - 3 = 0, \quad 0 \le x \le 2\pi.$

Sol'n If $k=\sin x$, considering $k^2+2k-3=0$

So, $(k+3)(k-1)=0$, factored left side.

$\therefore (\sin x+3)(\sin x-1)=0$, $k=\sin x$

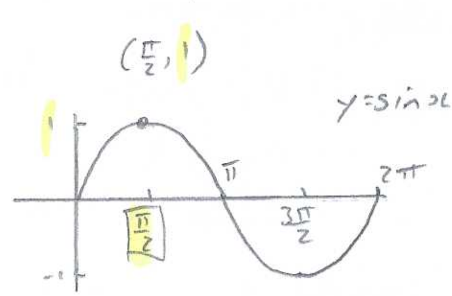
$\therefore \sin x+3=0$ or $\sin x-1=0$, logic

$\therefore \sin x=-3$ or $\sin x = 1$

$\therefore x=\sin^{-1}(-3)$ or $x=\sin^{-1}(1)$

no sol'n. $\quad x=\frac{\pi}{2}$

$\therefore$ Only real root is $\frac{\pi}{2}$ ✓

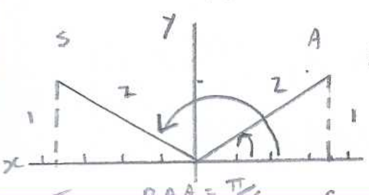


b) $2\sin^2 x+\sin x-1=0, \quad 0\le x\le 2\pi$

Sol'n $(2\sin x-1)(\sin x+1)=0$, Consider if $k=\sin x$, then factor $2k^2+k-1=0$
$(2k-1)(k+1)=0$

$\therefore 2\sin x-1=0$ or $\sin x+1=0$

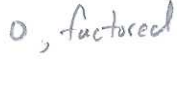
$\therefore \sin x=\frac{1}{2}$ $\quad \sin x=-1$



So $x_1=\frac{\pi}{6}$ ✓,

or $x_2=\frac{5\pi}{6}$ ✓,

So $x=\frac{3\pi}{2}$ ✓,



$\therefore$ 3 roots are $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ ✓✓✓

e)

$\underline{Ex_2}$ Solve $\cos^2 x = \frac{3}{4}$, $0 \le x \le 2\pi$. Present your answers correct to 2 decimal places.

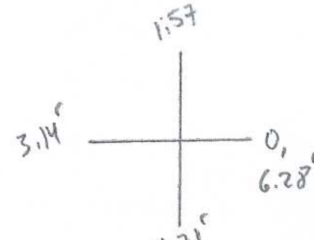
$\underline{Sol'n}$ Quadratic Trig Equation Can be thought of as $k^2 = \frac{3}{4}$ with $k = \cos x$,

$k = \pm\sqrt{\frac{3}{4}}$, undo square by rooting

$k = \pm\frac{\sqrt{3}}{\sqrt{4}}$

$k = \pm\frac{\sqrt{3}}{2}$

So $\cos x = \pm\frac{\sqrt{3}}{2}$.



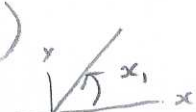
$\underline{\text{Case } \#1}$ or $\underline{\text{Case } \#2}$

Case #1

$\cos x = \frac{\sqrt{3}}{2}$

$x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$x_1 \doteq 0.5236^r$,



RAA $\doteq$ 0.52

or

$x_2 = 2\pi - RAA$

$= 2\pi - 0.5236$

$\doteq 5.76^r$

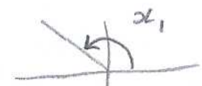
CAST pattern

Case #2

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$x_1 = 2.61799^r$,



RAA $= \pi - 2.62$

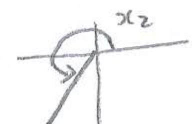
$\doteq 0.5198^r$

or

$x_2 = \pi + RAA$

$= \pi + 0.5198$

$\doteq 3.6614^r$



So $\underline{\text{Sol}}$ution $\underline{\text{Se}}$t aka S.S. $\doteq \{0.52^r, 2.62^r, 3.66^r, 5.76^r\}$

$\underline{Ex_3}$ Solve $\cos^2\theta + \cos\theta - 1 = 0$.

$\underline{Sol'n}$ $\underline{\text{Note}}$: Use quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ if solving the quadratic equation using the factor method presents a difficult for you factoring process. Generally if factoring is taking you longer than about 20 seconds, use quadratic formula.

Consider $k^2 + k - 1 = 0$, $k = \cos\theta$

$\boxed{\begin{array}{l} ax^2+bx+c=0 \\ a=1 \\ b=1 \\ c=-1 \end{array}}$

Roots: $k = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$k = \frac{-1 \pm \sqrt{(1)^2-4(1)(-1)}}{2(1)}$

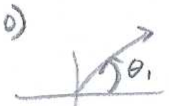
$k = \frac{-1 \pm \sqrt{5}}{2}$

So $\cos\theta = \frac{-1 \pm \sqrt{5}}{2}$ → $\frac{-1+\sqrt{5}}{2} \doteq 0.6180$, → $\frac{-1-\sqrt{5}}{2} \doteq -1.6180$

$\cos\theta \doteq 0.6180$ or $\cos\theta = -1.6180$
no sol'n

→ So $\theta = \cos^{-1}(0.6180)$

$\theta_1 \doteq 0.9046^r$,

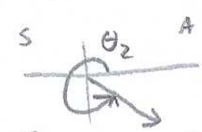


RAA $\doteq$ 0.9046

or

$\theta_2 = 2\pi - RAA$

$\doteq 5.3786^r$



$\therefore$ Roots are about $0.90^r, 5.38^r$, CAST patterns.

A p320 #(10,11) def, #14c