

# 4.4\* Finding the Vertex of A Parabola\*

Warmup:

1. Graph  $y = -2x^2 + 12x - 20$   
a            b            c

Sol'n:

$y = -2(x^2 - 6x) - 20$ , "a" to front  
 $y = -2(x^2 - 6x + 9 - 9) - 20$  special number  
 $y = -2(x^2 - 6x + 9) - 9(-2) - 20$ , pull  
 $y = -2(x - 3)^2 + 18 - 20$  factor  
 $y = -2(x - 3)^2 - 2$ , tidy up

Sol'n: Max occurs when  $x = -\frac{b}{2a}$

$x = -\frac{b}{2a}$  process  $x = -\frac{12}{2(-2)}$   
 $x = 3$

Max value is  $y = -2(3)^2 + 12(3) - 20$   
 $y = -2$

C.T.S. process

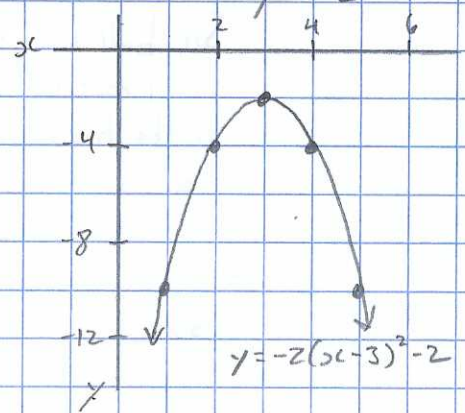
So, 1. Vertex (3, -2)

2. Mini-table of values:

x	y
2	-4
1	-10

"Sub and Solve"

3. Symmetry.



Ex, Determine the vertex for  $y = (x+3)(x-7)$ , a quadratic equation in factored form.

Sol'n First, FOIL to build Standard Form of the parabola.

$y = x^2 - 7x + 3x - 21$

$y = x^2 - 4x - 21$ , collected like terms.

Second, use either C.T.S. or  $-\frac{b}{2a}$  processes to find vertex

$-\frac{b}{2a}$

$a = 1$   
 $b = -4$   
 $c = -21$

Min when  $x = -\frac{(-4)}{2(1)} = 2$

Min is  $y = (2)^2 - 4(2) - 21 = -25$

V(2, -25)

C.T.S.

$y = x^2 - 4x - 21$

$y = x^2 - 4x + 4 - 4 - 21$

$y = (x - 2)^2 - 4 - 21$

$y = (x - 2)^2 - 25$

V(2, -25)

Ex<sub>2</sub> The path of a tennis ball can be modelled with the equation  $h = -0.06d^2 + 0.12d + 1$ , where  $h$  is the ball's height in meters and  $d$  is the horizontal distance from the player in meters.

a) What is the initial height of the tennis ball?

Sol'n Set  $d = 0$  and solve for  $h$

So,  $h = -0.06(0)^2 + 0.12(0) + 1$

$h = 1\text{m}$

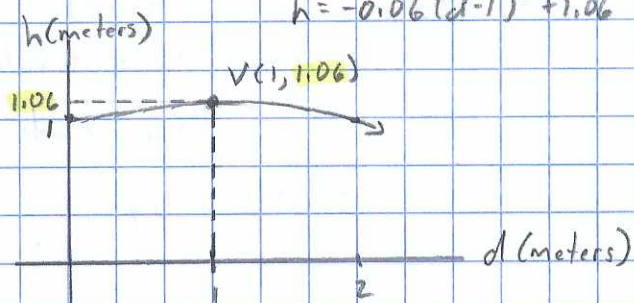
Thus ball is hit 1 meter above the ground by tennis racket. e<sub>s</sub>

b) What is the **max height** of the ball?

Sol<sup>n</sup>: Find the vertex using your favorite process.

$$\begin{aligned} \left(\frac{-b}{2a}\right) & a = -0.06 & \text{Max when } d = \frac{-b}{2a} = \frac{-(0.12)}{2(-0.06)} = 1 \text{ m (C.T.S.)} & h = -0.06d^2 + 0.12d + 1 \\ & b = 0.12 & & h = -0.06(d^2 - 2d) + 1, \text{ step \#1} \\ & c = 1 & \text{Max height is } h = -0.06(1)^2 + 0.12(1) + 1 & h = -0.06(d^2 - 2d + 1) + 1, \text{ step \#2} \\ & & = 1.06 \text{ m} & h = -0.06(d^2 - 2d + 1) - 1(-0.06) + 1 \\ & & & h = -0.06(d-1)^2 + 0.06 + 1 \\ & & & h = -0.06(d-1)^2 + 1.06 \end{aligned}$$

So, ball is **1.06 m** high when ball is 1 meter from the player. This is the highest height the tennis ball ever reaches on this shot.



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