$_N$TR.1 *Solving Trig Equations Using Grade 11 Identities*

Warmup: 1a) Solve $4\sqrt{2}\tan x - 11 = \sqrt{2}\tan x - 9$, $\pi \le x \le 2\pi$. b) Solve $2\cos\left(x + \frac{5\pi}{6}\right) + 4 = 3$, $0 \le x \le 2\pi$.

Sol'n $3\sqrt{2}\tan x = 2$

$\tan x = \frac{2}{3\sqrt{2}}$

$x_1 = 0.4405^r$, RAA

or

$x_2 = \pi + RAA$

$= 3.58^r$,

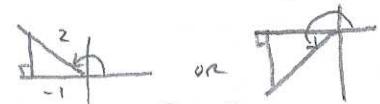
But 0.4405 inadmissible since it is too small.

$\pi \le x \le 2\pi$. ($\pi \doteq 3.14^r$, $2\pi \doteq 6.28^r$) So $x \doteq 3.58^r$. ✓

Sol'n Let $A = x + \frac{5\pi}{6}$.

So $2\cos A + 4 = 3$

$\cos A = -\frac{1}{2}$ ← must be a special angle!



RAA $= \frac{\pi}{3}$.

So $A = \frac{2\pi}{3}$ or $A = \frac{4\pi}{3}$

$\therefore x + \frac{5\pi}{6} = \frac{2\pi}{3}$

$x = -\frac{\pi}{6}$

$x = \frac{11\pi}{6}$, coterminal angle to $-\frac{\pi}{6}$. ✓

$\therefore x + \frac{5\pi}{6} = \frac{4\pi}{3}$, back sub

$x = \frac{3\pi}{6} = \frac{\pi}{2}$ ✓

Remark$_1$: We usually substitute established identities to try to express the equation in terms of a single trig function. Additionally, use algebraic skills and your ingenuity. For example, if we can build a product of factors aka $(\quad)(\quad) = 0$, then we can solve by Zero Product Property.

Remark$_2$: Don't divide away some of the roots. Recall: If $x^2 + x = 0$, then $x(x+1) = 0$, factor x to front... don't divide it away to zero. and thus $x = 0$ or $x = -1$.

Ex$_1$ Solve $2\sin x = -\cos x$, $0° \le x \le 360°$.

Sol'n $\frac{2\sin x}{\cos x} = \frac{-\cos x}{\cos x}$, divided by $\cos x$ and we comment $\cos x \ne 0$.

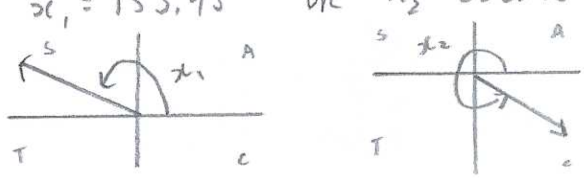
$2\tan x = -1$, tangent identity sub.

$\tan x = -\frac{1}{2}$

$\therefore x \doteq -26.5651°$, calculator

$\therefore$ RAA $\doteq 26.5651°$

$\therefore x_1 = 153.43°$ or $x_2 = 333.43°$



Ex_2 Find all p.o.i between $y = 2\sin^2 x$ and $y = 3 - 3\cos x$, $0 \le x \le 2\pi$.

Sol^n At point of intersection, $y = y$

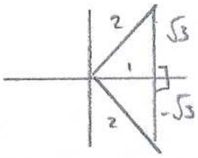
$\therefore \quad 2\sin^2 x = 3 - 3\cos x.$

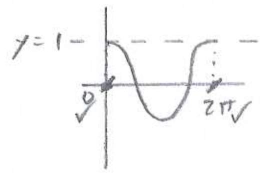
$2(1-\cos^2 x) = 3 - 3\cos x$, twisted pythag sub

$2 - 2\cos^2 x = 3 - 3\cos x$

$0 = 2\cos^2 x - 3\cos x + 1$

$0 = (2\cos x - 1)(\cos x - 1)$, built product of factors ✓





$\therefore \cos x = \frac{1}{2}$ or $\cos x = 1$

$\therefore x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad x = 0, 2\pi$

So Answer Set is $\{0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi\}$

$Remark_3$: In general, not always, when more than one trig function (sin, cos, tan, csc, sec, cot) is involved, make a substitution/manipulation to try to reduce the equation to a single trig function.

Ex_3 Solve $2\cos\theta - \cot\theta = 0$, $0 \le \theta \le 2\pi$

Sol^n

$2\cos\theta - \frac{\cos\theta}{\sin\theta} = 0$, sub $\cot\theta$ out.

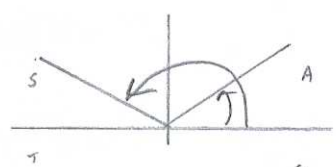
$\frac{2\cos\theta\sin\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \frac{0}{\sin\theta}$, built common denom

$2\cos\theta\sin\theta - \cos\theta = 0$, clear denom

$\cos\theta(2\sin\theta - 1) = 0$, common factor $\cos\theta$ to front, built product of factors ✓

$\therefore \cos\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$



$RAA = \frac{\pi}{6}$

$Note_1$: This solution nicely illustrates a successful process of not expressing equation in terms of a single trig function. As long as you can build (X) = 0, you can solve it.

| W.S. #2,3,4 |

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Gizmo: "Simplifying Trig Expressions"

Trig Equations with Grade 11 Identities

1. In each of the following, use a substitution for $\tan\theta$ or $\cot\theta$ in order to simplify. Solve the resulting equation for $0 \le \theta \le 2\pi$.
   a) $2\cos\theta - \cot\theta = 0$
   b) $2\sin\theta\tan\theta = 5 - \dfrac{1}{\cos\theta}$

2. Solve each of the following for $0 \le x \le 2\pi$.
   a) $\cos^2 x + \sin x + 1 = 0$
   b) $4\sin^2 x + \sin x + 3 = 6\cos^2 x$

3. Find the points of intersection of the functions $f(x) = \sin^2 x - 3\cos^2 x$ and $g(x) = 8\cos x - 4$, $0 \le x \le 2\pi$.

4. Solve the equation for $-\pi \le x \le 2\pi$. $6\cos x + 5\tan x = 0$

5. Solve the following equation for $0 \le x \le 2\pi$. $2\sin^2 x = 1 - \cos x$.

Ans: 1. a) $\dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$ b) $\dfrac{\pi}{3}, \dfrac{5\pi}{3}$ 2. a) $\dfrac{3\pi}{2}$ b) $3.79^r, 5.64^r, \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

3. $\left(\dfrac{\pi}{3}, 0\right), \left(\dfrac{5\pi}{3}, 0\right)$ 4. $-2.41^r, -0.73^r, 3.87^r, 5.55^r$ 5. $0, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, 2\pi$

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