Co-Functions

Warmup*: 1. Solve $2\cos x + 3\tan x = 0$, for $0° \le x \le 360°$.

Soln Recall*: In general, not always, since more than one trig function is involved, make a substitution / manipulation or two to try to reduce the given equation to a single trig function.

$2\cos x + 3\frac{\sin x}{\cos x} = 0$, sub $\tan$ I.D., $\cos x \neq 0 \Rightarrow x \neq 90°, 270°$

$\frac{2\cos^2 x + 3\sin x}{\cos x} = \frac{0}{\cos x}$, built common denom

$2\cos^2 x + 3\sin x = 0$, clear common denom.

$2(1-\sin^2 x) + 3\sin x = 0$, sub twisted pythagorean I.D.

$-2\sin^2 x + 2 + 3\sin x = 0$

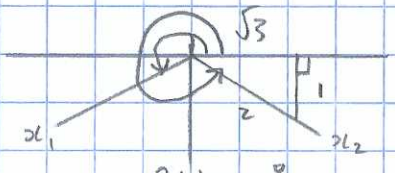
$2\sin^2 x - 3\sin x - 2 = 0$, rearrange and multiply by -1.

$(2\sin x + 1)(\sin x - 2) = 0$, consider $2k^2 - 3k - 2 = 0$

$\therefore \sin x = -\frac{1}{2}$ or $\sin x = 2$ $\quad (2k+1)(k-2) = 0$

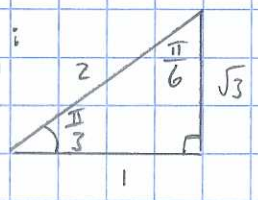
no roots.

$x = \sin^{-1}(-\frac{1}{2})$



$\therefore x_1 = 210°$ and $x_2 = 330°$ //

Consider$_1$:



i) $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

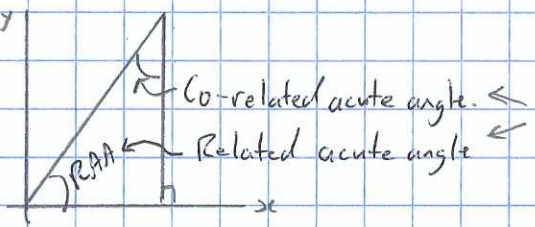
ii) $\tan\frac{\pi}{3} = \frac{\sqrt{3}}{1}$ $\quad \cot\frac{\pi}{6} = \frac{\sqrt{3}}{1}$

$\therefore \sin\frac{\pi}{3} = \cos\frac{\pi}{6}$

Sum $= \frac{\pi}{2}$

$\tan\frac{\pi}{3} = \cot\frac{\pi}{6}$

Consider$_2$*:



Sum $= \frac{\pi}{2}$.

Sum $= \frac{\pi}{2} = 90°$

Summary*: Co-functions have equal trig ratio values if their angles are complementary.

6 co-function identities*: $\sin\theta = \cos(\frac{\pi}{2} - \theta)$; $\tan\theta = \cot(\frac{\pi}{2} - \theta)$; $\sec\theta = \csc(\frac{\pi}{2} - \theta)$

$\cos\theta = \sin(\frac{\pi}{2} - \theta)$; $\cot\theta = \tan(\frac{\pi}{2} - \theta)$; $\csc\theta = \sec(\frac{\pi}{2} - \theta)$

"sine tangent secant" "3 couples"
"cosine cotangent cosecant"

Ex 1 Express $\sin\frac{5\pi}{12}$ in terms of its cofunction.

Soln $= \cos\left(\frac{\pi}{2} - \frac{5\pi}{12}\right)$, sub in cofunction and sub in $90° = \frac{\pi}{2}$ - given angle.

$= \cos\left(\frac{6\pi}{12} - \frac{5\pi}{12}\right)$, common denom

$= \cos\frac{\pi}{12}$.

Ex 2 Express $\csc\frac{8\pi}{17}$ in terms of its cofunction

Soln $= \sec\left(\frac{\pi}{2} - \frac{8\pi}{17}\right)$

↑ cofunction ← Complementary Angle

$= \sec\left(\frac{17\pi}{34} - \frac{16\pi}{34}\right)$

$= \sec\frac{\pi}{34}$

Check: LS $= \csc\frac{8\pi}{17}$

$= \frac{1}{\left(\sin\frac{8\pi}{17}\right)}$, by def'n

$\doteq 1.004$, calculator

RS $= \sec\frac{\pi}{34}$

$= \frac{1}{\left(\cos\frac{\pi}{34}\right)}$, by def'n

$\doteq 1.004$, calculator

$\therefore$ LS $\doteq$ RS

$\therefore \csc\frac{8\pi}{17} = \sec\frac{\pi}{34}$.

W.S. # 1-5,
" 7d
" 8d