4.2 Quadratic Functions

The word quadratic comes from the Latin word quadratum, meaning square. In a quadratic function one of the terms contains the independent variable raised to the exponent 2.

ex. $y = 4x^2$ Quadratic $y = x^2 + 3$ Quadratic

$y = 3x^2 + 2x + 1$ Quadratic $y = 2x + 3$ Linear Function

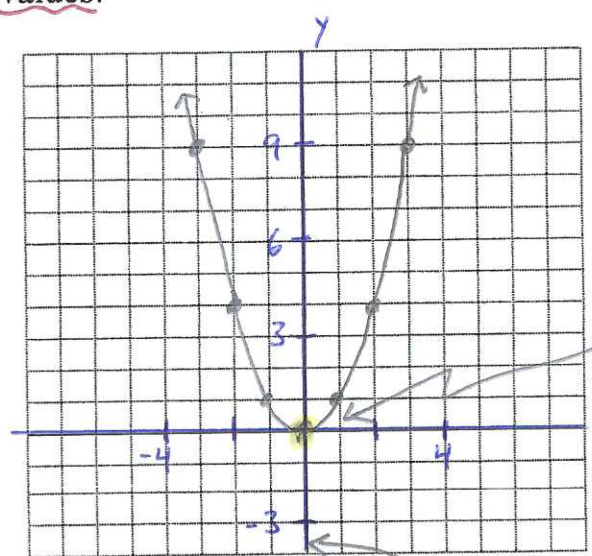
The simplest quadratic function is $y = x^2$. In function notation : $f(x) = x^2$, where f is the squaring operation.

input

Graph this function using a table of values:

Plug in

x	$y = (x)^2$
-3	$= (-3)^2 = 9$
-2	$= (-2)^2 = 4$
-1	$= (-1)^2 = 1$
0	$= 0$
1	$= 1$
2	$= 4$
3	$= 9$



$y = x^2$

Vertex aka Tip

Axis of Symmetry: Line $x = 0$ aka the y-axis.

The shape of this graph is called parabolic.

Parabolas have a geometric property called symmetry. Symmetrical figures are those that could be folded along a fold line so that one half of the figure exactly matches the other half. This fold line is called the axis of symmetry. The vertex of the parabola is the point where the graph intersects the axis of symmetry.

"k" is a number. "k" is a number.

Functions of the form $y = x^2 + k$. (In function notation: $y = f(x) + k$ **)**

We will look at transformations of the basic function $y = x^2$. First, we will investigate the effect of adding a value **k** to get $y = x^2 + k$.

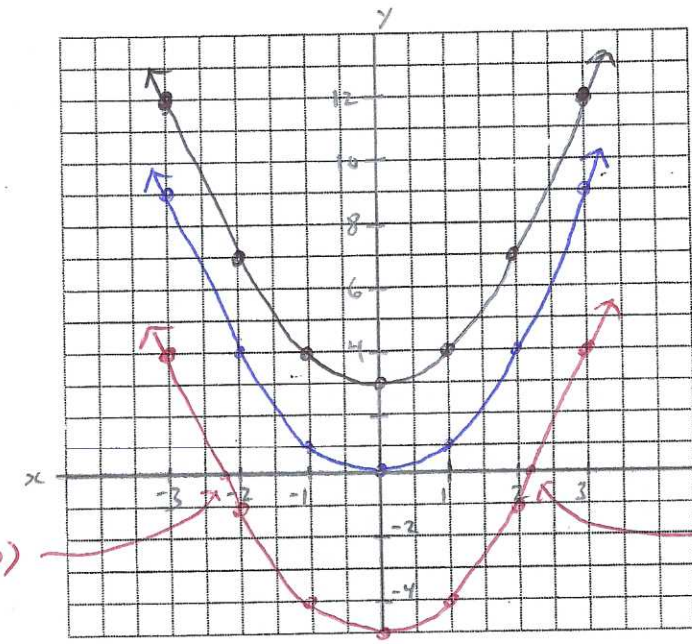
1. $y = x^2$
2. $y = x^2 + 3$
3. $y = x^2 - 2$
4. $y = x^2 - 5$
5. $y = x^2 + 4$

x	$y = x^2$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

x	$y = x^2 + 3$
-3	$= (-3)^2 + 3 = 12$
-2	$= (-2)^2 + 3 = 7$
-1	4
0	3
1	4
2	7
3	12

x	$y = x^2 - 5$
-3	$= (-3)^2 - 5 = 4$
-2	$= (-2)^2 - 5 = -1$
-1	$= -4$
0	-5
1	-4
2	-1
3	4

Graph each of the functions on the grid below. Label each parabola with its equation.



$y = x^2 + 3$

$y = x^2$

$y = x^2 - 5$

x-intercept $\doteq (2.2, 0)$

x-intercept $\doteq (-2.2, 0)$

How are the graphs different? How are they the same?

Different Vertices

Different Places On Grid

Same slope since same shape and same size. (aka Congruent.)

Same axis of symmetry.

What is the effect of adding a value of k to the basic function to get $y = x^2 + k$?

* Summary *

* If k is greater than zero, aka, if $k > 0$, then $y = x^2$ is translated Up "k" units.

* If k is less than zero, aka, if $k < 0$, then $y = x^2$ is translated Down "k" units.

For each of the following identify the vertex, axis of symmetry, x-intercepts, maximum or minimum value, domain, and range.

1. $y = x^2 + 3$

vertex: $(0, 3)$
axis of symmetry: $x = 0$
x-intercepts: None
max/min value: Minimum $y = 3$
domain: {all x}
range: {all y, $y \geq 3$}

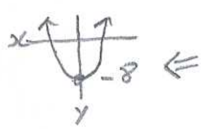
"y greater than or equal to 3"

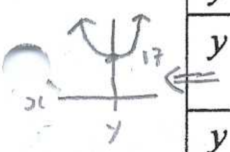
2. $y = x^2 - 5$

vertex: $(0, -5)$
axis of symmetry: $x = 0$
x-intercepts: About ± 2.2
max/min value: $y = -5$
domain: {all x}
range: {all y, $y \geq -5$}

Without graphing, fill in the following chart for each of the functions:

"Sketchy Sketchy"





Function	Vertex	Axis of Symmetry	Max./Min. Value	x-intercepts *	Domain	Range
$y = x^2 - 8$	$(0, -8)$	$x = 0$	Min $y = -8$	$(\pm\sqrt{8}, 0)$	{all x}	{all y, $y \geq -8$}
$y = x^2 + 1$						
$y = x^2 + 17$	$(0, 17)$	$x = 0$	Min $y = 17$	None	{all x}	{all y, $y \geq 17$}
$y = x^2 - 23$						

* To find the x-intercepts, set $y = 0$ and solve for x.

For $y = x^2 - 8$.

x-intercepts:
Set $y = 0$. Isolate x.

$0 = x^2 - 8$
$+8 \qquad +8$

$8 = x^2$
$\sqrt{8} = \sqrt{x^2}$
$x = \pm\sqrt{8}$

A side: Undo Order: SAMDEB

P213 # 1abg, 2a, 6ab

Functions in the form $y = ax^2$. In function notation: ____________.
Now we will look at the effect of multiplying by a value a to get $y = ax^2$.
Graph each of the following on the TI-83+.

1. $y = x^2$
2. $y = 2x^2$
3. $y = 3x^2$
4. $y = \frac{1}{2}x^2$
5. $y = \frac{1}{3}x^2$