TR.2 N

* Addition Identities & Sine and Cosine *

Warmup: 1. Write in terms of the cofunction identity.

a) $\sin \frac{3\pi}{16}$

Sol'n $= \cos\left(\frac{8\pi}{16} - \frac{3\pi}{16}\right)$

$= \cos\left(\frac{5\pi}{16}\right)$

b) $\cot \frac{3\pi}{4}$

Sol'n $= \tan\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)$

$= \tan\left(\frac{2\pi}{4} - \frac{3\pi}{4}\right)$

$= \tan\left(\frac{-\pi}{4}\right)$

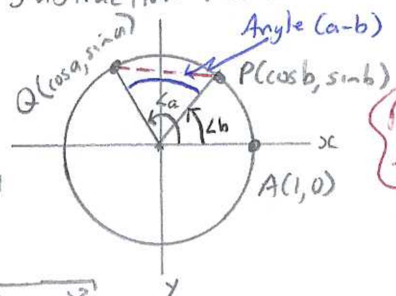
c) $\sec \frac{3\pi}{4}$

Sol'n $= \csc\left(\frac{2\pi}{4} - \frac{3\pi}{4}\right)$

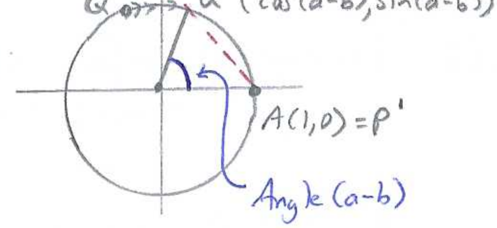
$= \csc\left(\frac{-\pi}{4}\right)$

Remark₁: Subtraction Formula for Cosine: $\boxed{\cos(a-b) = \cos a\cos b + \sin a\sin b}$

Proof:



Rotate Point P to Point A



---- "rope to tow"

✓ Unit Circle: r=1

✓ Sym (x, T, y, x.

✓ $L = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

∴ Length of PQ "Sector Angle = a−b" = Length of AQ' "Sector Angle = a−b"

$\therefore \left(\sqrt{(\cos a - \cos b)^2 + (\sin a - \sin b)^2}\right)^2 = \left(\sqrt{(\cos(a-b)-1)^2 + (\sin(a-b)-0)^2}\right)^2$, Length formula

$(\cos a - \cos b)^2 + (\sin a - \sin b)^2 = (\cos(a-b)-1)^2 + (\sin(a-b))^2$

$\underbrace{\cos^2 a + \cos^2 b + \sin^2 a + \sin^2 b}_{=1,\ =1} - 2\cos a\cos b - 2\sin a\sin b = \underbrace{\cos^2(a-b) + \sin^2(a-b)}_{=1} - 2\cos(a-b) + 1$, FOIL + collect likes

$2 - 2\cos a\cos b - 2\sin a\sin b = 2 - 2\cos(a-b)$

$-2 \qquad\qquad -2$

$-2\cos a\cos b - 2\sin a\sin b = -2\cos(a-b)$

$\cos a\cos b + \sin a\sin b = \cos(a-b)$, Divided by −2.

So $\boxed{\cos(a-b) = \cos a\cos b + \sin a\sin b}$

"Cos of a Difference stretches to "cos-cos-sin-sin-sum"!"

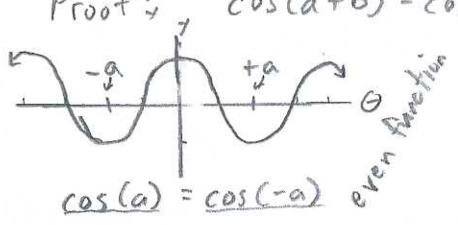
Remark₂: Addition Formula for Cosine: $\boxed{\cos(a+b) = \cos a\cos b - \sin a\sin b}$

Proof: $\cos(a+b) = \cos(a-(-b)) = \cos a\cos(-b) + \sin a\sin(-b)$

$= \cos a\cos b + \sin a(-\sin b)$

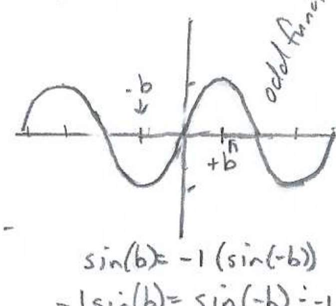
$\therefore \boxed{\cos(a+b) = \cos a\cos b - \sin a\sin b}$

"cos of a Sum stretches to "cos-cos-sin sin-difference"!



$\cos(a) = \cos(-a)$ even function

Equal outputs aka function values aka y-values



$\sin(b) = -1(\sin(-b))$

$-1\sin(b) = \sin(-b)$; ÷ −1

es

Remark$_3$: Addition Formula for Sine: $\boxed{\sin(a+b) = \sin a \sin b + \cos a \cos b}$

Proof: $\sin(a+b) = \cos\left(\frac{\pi}{2} - (a+b)\right)$, cofunction identity: Cofunctions have equal ratio values if their angles are complementary So $\cos(90° - 30°) = \sin 30°$

$= \cos\left(\left(\frac{\pi}{2} - a\right) - b\right)$, distribution.

$= \cos\left(\frac{\pi}{2} - a\right)\cos(b) + \sin\left(\frac{\pi}{2} - a\right)\sin(b)$, cosine difference identity

"sin of a sum gets stretched out" $\boxed{\sin(a+b) = \sin a \cos b + \cos a \sin b}$, cofunction identity for 2 substitutions.

Remark$_4$: Subtraction Formula for Sine: $\boxed{\sin(a-b) = \sin a \cos b - \sin b \cos a}$

Proof: $\sin(a-b) = \sin(a + (-b))$

$= \sin a \underline{\cos(-b)} + \underline{\sin(-b)} \cos a$, sin (Sum) stretched.

$= \sin a \underline{\cos b} + \underline{(-1)\sin b} \cos a$, double substitution of even and odd functions.

"sin of a difference stretched out" $\boxed{\sin(a-b) = \sin a \cos b - \sin b \cos a}$, tidy up.

Ex$_1$ Find the exact value of $\sin 50° \cos 20° - \cos 50° \sin 20°$

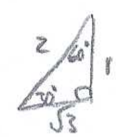
Sol'n Using patterning and comparing to the above 4 addition and subtraction identities,

$= \sin(50° - 20°)$, sin (Diff) identity

$= \sin 30°$

$= \frac{1}{2}$, 

Ex$_2$ Calculate $\sin\left(\frac{7\pi}{12}\right)$ exactly.



Sol'n *Tip: Add and Subtract Special Angles to Obtain $\frac{7\pi}{12} = 105°$ *

$\sin \frac{7\pi}{12} = \sin 105° = \sin(45° + 60°) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$ ✓ $\rightarrow = \frac{\sqrt{2}+\sqrt{6}}{4}$ rationalized denom ✓

$= \frac{(1+\sqrt{3})}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

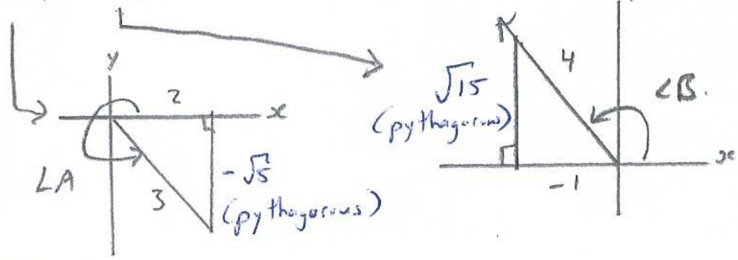
Note$_1$: $\left.\begin{aligned} \sin 105° &= \sin(135° - 30°) \\ &= \sin(150° - 45°) \\ &= \sin(225 - 120°) \end{aligned}\right\}$ Any of these stretched out would have worked as well ☺

e_3

$\underline{Ex_3}$ If $\cos\angle A = \frac{2}{3}$, $\frac{3\pi}{2} < A \le 2\pi$, and $\cos\angle B = -\frac{1}{4}$, $\frac{\pi}{2} < B < \pi$, find $\cos(A+B)$.

Sol'n $\cos(A+B) = \cos A \cos B - \sin A \sin B$, cos (sum) stretch

$= \left(\frac{2}{3}\right)\left(\frac{-1}{4}\right) - (\quad)(\quad)$, sub in 2 given ratios.



$= \left(\frac{2}{3}\right)\left(\frac{-1}{4}\right) - \left(\frac{-\sqrt{5}}{3}\right)\left(\frac{\sqrt{15}}{4}\right)$

$= \frac{-2}{12} + \frac{\sqrt{75}}{12}$

$= \frac{-2}{12} + \frac{\sqrt{25 \cdot 3}}{12}$

$= \frac{-2}{12} + \frac{5\sqrt{3}}{12}$ ✓

W.S. Compound Angles - Pg 1 - Answers on Pg 1 Bottom - Every Other Letter.

Gizmo #2 For Unit #6's "Sum and Difference Identities for Sine and Cosine"