4.4 * Completing the Perfect Square Trinomial *
aka C.T.S.

Objective: Rewrite standard form $y=ax^2+bx+c$ into vertex form $y=a(x-h)^2+k$.

Warmup: 1. Factor i) $x^2+12x+36$ ii) $x^2+10x+25$ iii) x^2-6x+9

"constant term" "2ND degree term" "1st degree term"

"Used sum and product trinomial factoring"

i) $= (x+6)(x+6)$
$= (x+6)^2$

ii) $= (x+5)(x+5)$
$= (x+5)^2$

iii) $= (x-3)(x-3)$
$= (x-3)^2$

Pattern: The constant term is the square of half the 1st degree term.

Ex1 Find "k" to complete the square.

i) $x^2+14x+k$
Sol'n $= (x+7)^2$, $k=(7)^2 = 49$ ✓

ii) x^2+8x+k
Sol'n $= (x+4)^2$, $k=(4)^2 = 16$ ✓

iii) $x^2-16x+k$
Sol'n $=(x-8)^2$, $k=64$ ✓

Ex2 Rewrite a) $y=x^2+8x+27$ into vertex form $y=a(x-h)^2+k$, ← "easy to graph" ← "easy to find min/max y-value."

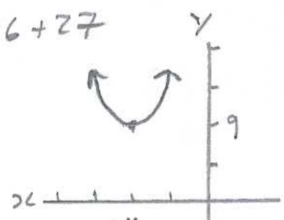
Sol'n ✓ Look at terms containing x^2 and x. $y=\underline{x^2+8x}+27$

✓ Complete the Square: Ask yourself, what number must I add to C.T.S.? This special number should be added and subtracted so you don't change the equation.

$y=\underline{x^2+8x+16}-16+27$
$+\left(\frac{8}{2}\right)^2-\left(\frac{8}{2}\right)^2=0$

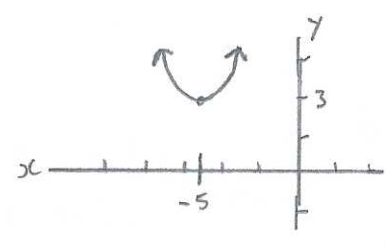
✓ Sum and Product Trinomial factoring: $y=\underline{(x+4)^2}-16+27$

✓ Tidy up: $y=(x+4)^2+9$



This parabola has a minimum of y=11 that occurs when $x=-4$. This parabola has vertex $(-4,9)$.

b) $y=\underline{x^2+10x}+28$



Sol'n $y=\underline{x^2+10x+25}-25+28$, Completed the build of the perfect square trinomial by adding and subtracting the special number 25.
$+\left(\frac{10}{2}\right)^2-\left(\frac{10}{2}\right)^2$

$y=\underline{(x+5)^2}-25+28$
factored

$y=(x+5)^2+3$, tidy up max/min value

This parabola has a minimum value when $x=-5$
Min. value is $y=3$

Remark$_1$: The 4 steps to convert to vertex form by "completing the square."

Step 1: Factor "a" out of the first two terms

Step 2: Add and subtract the special number.

Step 3: Pull the special number out of the bracket and multiply by "a"

Step 4: Factor the perfect square trinomial and tidy up min/max value.

Note$_1$: The following 2 examples are as challenging as C.T.S. gets. So here we go!

Ex$_3$ Rewrite a) $y = \underline{4x^2 + 24x} + 38$ into vertex form $y = a(x-p)^2 + q$ using our C.T.S. plan.

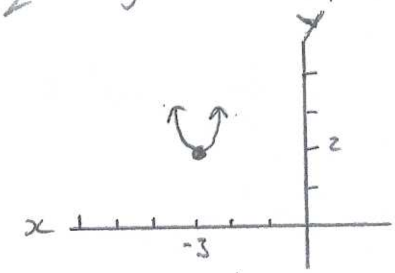
Sol'n Step 1: $y = \underline{4(x^2 + 6x)} + 38$

Step 2: $y = \underline{4(x^2 + 6x + 9 - 9)} + 38$, C.T.S. with $\left(\frac{6}{2}\right)^2 = 9$

Step 3: $y = \underline{4(x^2 + 6x + 9)} - 9(4) + 38$, pull and multiply

Step 4: $y = \underline{4(x+3)^2} - 9(4) + 38$, factor trinomial

$y = 4(x+3)^2 + 2$, tidy up.



Min occurs when $x = -3$

Min value is $y = 2$.

b) $y = \underline{-3x^2 + 48x} - 300$

Sol'n

#1 $y = -3(x^2 - 16x) - 300$, common factor -3 to front

#2 $y = -3(x^2 - 16x + 64 - 64) - 300$, C.T.S.

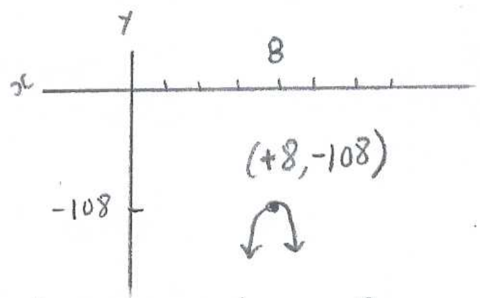
$+\left(\frac{16}{2}\right)^2 - \left(\frac{16}{2}\right)^2$

"This is the challenging" step... →

#3 $y = -3(x^2 - 16x + 64) - 64(-3) - 300$, pulled -64 out of bracket and multiply by "a"

#4 $y = -3(x-8)^2 - 64(-3) - 300$, factored

$y = -3(x-8)^2 + 192 - 300$
$y = -3(x-8)^2 - 108$ } tidy up.



Max when $x = 8$

Max is $y = -108$.

Remark$_2$: Lots of potential for error when following this C.T.S. process. That said, some staff and students find the C.T.S. process quite efficient!

p235 #4b, #7ade