Compound Angles

A. $\cos(x+y)=\cos x\cos y-\sin x\sin y$ $\qquad$ $\cos(x-y)=\cos x\cos y+\sin x\sin y$

1. Express each of the following as a single trigonometric function
   a) $\cos 2x\cos x-\sin 2x\sin x$ $\qquad$ b) $\cos 4a\cos 4a-\sin 4a\sin 4a$
   c) $\cos(x+2)\cos(y+2)+\sin(x+2)\sin(y+2)$ $\qquad$ d) $\cos^2 2x+\sin^2 2x$

2. For each of the following use the sum or difference formula to simplify.
   a) $\cos\left(\frac{\pi}{2}-x\right)$ b) $\cos\left(x-\frac{\pi}{2}\right)$ c) $\cos(\pi-x)$ d) $\cos(x-\pi)$

3. Evaluate exactly. a) $\cos\frac{5\pi}{12}$ b) $\cos\frac{\pi}{12}$ c) $\cos\frac{7\pi}{12}$ d) $\cos\left(\frac{\pi}{3}+\frac{\pi}{2}\right)$

4. If $0\le a\le\frac{\pi}{2}$ and $\frac{3\pi}{2}\le b\le 2\pi$ and $\cos a=\frac{4}{5}$, $\cos b=\frac{3}{5}$, find $\cos(a-b)$ and $\cos(a+b)$.

5. If $\sec x=\frac{17}{8}$ and $\csc y=\frac{5}{3}$ where x and y lie in the first quadrant, find $\sec(x+y)$.

B. $\sin(x+y)=\sin x\cos y+\sin y\cos x$ $\qquad$ $\sin(x-y)=\sin x\cos y-\sin y\cos x$

1. Express as a single trigonometric function.
   a) $\sin 3x\cos x+\cos 3x\sin x$ $\qquad$ b) $\sin 5a\cos 2a-\cos 5a\sin 2a$
   c) $\sin 2y\cos y+\cos 2y\sin y$ $\qquad$ d) $\sin(x+3)\cos(y+3)-\cos(x+3)\sin(y+3)$
2. For each of the following use the sum or difference formula to simplify.
   a) $\sin\left(\frac{\pi}{2}-x\right)$ b) $\sin\left(x-\frac{\pi}{2}\right)$ c) $\sin(\pi-x)$ d) $\sin(x-\pi)$

3. Evaluate exactly. a) $\sin\frac{5\pi}{12}$ b) $\sin\frac{\pi}{12}$ c) $\sin\frac{7\pi}{12}$ d) $\sin\left(\frac{\pi}{3}+\frac{\pi}{2}\right)$

4. Given $\sin x=\frac{3}{5}$ and $\sin y=\frac{8}{17}$, $\frac{\pi}{2}\le x\,and\,y\le\pi$, find $\sin(x+y)$, $\sin(x-y)$ and $\sin(y-x)$.

5. If $\sec x=\frac{17}{8}$ and $\csc y=\frac{13}{5}$ where x and y lie in the first quadrant, find $\csc(x+y)$.

Answers

Cosine 1. a) $\cos 3x$ b) $\cos 8a$ c) $\cos(x-y)$ d) 1 2. a) $\sin x$ b) $\sin x$ c) $-\cos x$ d) $-\cos x$

3. a) $\frac{\sqrt{6}-\sqrt{2}}{4}$ b) $\frac{\sqrt{6}+\sqrt{2}}{4}$ c) $\frac{\sqrt{2}-\sqrt{6}}{4}$ d) $-\frac{\sqrt{3}}{2}$ 4. 0, $\frac{24}{25}$ 5. $-\frac{85}{13}$

Sine 1. a) $\sin 4x$ b) $\sin 3a$ c) $\sin 3y$ d) $\sin(x-y)$ 2. a) $\cos x$ b) $-\cos x$ c) $\sin x$ d) $-\sin x$

3. a) $\frac{\sqrt{6}+\sqrt{2}}{4}$ b) $\frac{\sqrt{6}-\sqrt{2}}{4}$ c) $\frac{\sqrt{6}+\sqrt{2}}{4}$ d) $\frac{1}{2}$ 4. a) $-\frac{77}{85}$ b) $-\frac{13}{85}$ c) $\frac{13}{85}$ 5. $\frac{221}{220}$

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C. For all $x, y \in R$ where x, y, $(x+y)$ and $(x-y)$ are not odd multiples of $\frac{\pi}{2}$,

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y} \qquad \tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$$

1. Express as a single trig function. a) $\frac{\tan 2a+\tan a}{1-\tan 2a\tan a}$ b) $\frac{\tan 5x-\tan 2x}{1+\tan 5x\tan 2x}$ c) $\frac{\tan x+\tan x}{1-\tan^2 x}$

2. Evaluate exactly. a) $\tan\frac{5\pi}{12}$ b) $\tan\frac{\pi}{12}$ c) $\tan\frac{7\pi}{12}$ d) $\tan(\frac{\pi}{3}+\pi)$

3. Explain why the formula for $\tan(x+y)$ cannot be applied to $\tan\left(\frac{\pi}{2}+x\right)$.

4. Show that $\cot(x+y)=\frac{\cot x\cot y-1}{\cot x+\cot y}$. Recall that $\cot x=\frac{\cos x}{\sin x}$.

5. Given $\tan x=\frac{3}{4}$ and $\tan y=\frac{12}{5}$, where x and y lie in the first quadrant, find $\tan(x+y)$, $\tan(x-y)$ and $\tan(y-x)$.

6. If $\tan x=\frac{4}{3}$ and $\cos y=-\frac{8}{17}$, where $\pi\le x, y\le\frac{3\pi}{2}$, find $\tan(x-y)$.

Answers

Tangent 1. a) $\tan 3a$ b) $\tan 3x$ c) $\tan 2x$ 2. a) $2+\sqrt{3}$ b) $2-\sqrt{3}$ c) $-2-\sqrt{3}$ d) $\sqrt{3}$

3. $\tan\frac{\pi}{2}$ is undefined 4. See derivation of tan(x+y) 5. $\frac{-63}{16}$, $-\frac{33}{56}$, $\frac{33}{56}$ 6. $-\frac{13}{84}$