TR.1 * Trig Identities *

↳ "Use skill + strategy to determine if trig equation is true for all values of the variable"

WARMUP:

Solve, $0 \le \theta \le 2\pi$

a) $2\cos\theta + 1 = 0$

$\cos\theta = \frac{-1}{2}$

$\theta_1 = \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$\theta_2 = \pi + \frac{\pi}{3}$

$\theta_2 = \frac{4}{3}\pi$

b) $\sqrt{3}\tan\theta = 1$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta_1 = \frac{\pi}{6}$

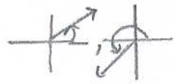
$\theta_2 = \pi + \frac{\pi}{6}$

$\theta_2 = \frac{7}{6}\pi$

c) $\frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$, divided both sides by $\cos\theta$.

$\tan\theta = 1$, ah ha! From 3U, left side $\frac{\sin\theta}{\cos\theta} = \tan\theta$.

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

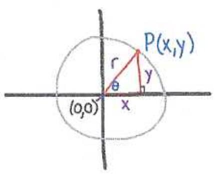


, $RAA = \frac{\pi}{4}$

An identity is an equation which is true for all values of the variables

An example of an algebraic identity is $x^2 - 9 = (x+3)(x-3)$

Eg$_1$ Show that $\boxed{\cos^2\theta + \sin^2\theta = 1}$ Pythagorean Identity



$\cos\theta = \frac{x}{r}$; $\sin\theta = \frac{y}{r}$

$r\cos\theta = x$; $r\sin\theta = y$, multiply by r

$\because x^2 + y^2 = r^2$

$\therefore (r\cos\theta)^2 + (r\sin\theta)^2 = r^2$

$\cancel{r^2}\cos^2\theta + \cancel{r^2}\sin^2\theta = \cancel{r^2}$, $\div r^2$

$\therefore \cos^2\theta + \sin^2\theta = 1$, $\div \cos^2\theta$

$1 + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\boxed{1 + \tan^2\theta = \sec^2\theta}$

$\div \sin^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta} + 1 = \frac{1}{\sin^2\theta}$

$\boxed{\cot^2\theta + 1 = \csc^2\theta}$

Eg$_2$ Show that $\boxed{\tan = \frac{\sin\theta}{\cos\theta}}$ Tan Identity.

r = 1

Consider the unit circle, when P(x,y) is on the terminal arm of θ and on the circumference of the unit circle. We know, from example 1, $x = (1)\cos\theta$ and $y = (1)\sin\theta$. $\therefore P(x,y) = (\cos\theta, \sin\theta)$. Now $\tan\theta = \frac{y}{x}$.

Thus $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Algebraic skills to prove identities

1. Change all trig ratios to sin and cosine
2. Factor when available
3. Sub using identies above
4. Common denominator

Strategies in making a formal proof

1. Choose one side, manipulate til both sides look the same
2. Choose most complex side
3. Work on both sides simultaneously

Ex. Prove these identities. [Tip: Use skills and strategies listed above, together with 4 boxed trig identities above, to prove Left Side = Right Side. NEVER cross the equal sign. Work on each side alone until you have L.S. = R.S.]

a) $\dfrac{\sin^2\theta}{\tan^2\theta} = 1-\sin^2\theta$

Proof

"aka Test aka Tempt"

L.S. $= \dfrac{\sin^2\theta}{\left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)}$, sub

$= \sin^2\theta \times \dfrac{\cos^2\theta}{\sin^2\theta}$, multiply by reciprocal

$= \cos^2\theta$, divided common factor to 1

R.S. $= (\cos^2\theta + \sin^2\theta) - \sin^2\theta$, sub using Pythagorean Identity.

$= \cos^2\theta$, collected like terms

$\because$ LS = RS

$\therefore$ Proven.

b) $1+\cos\theta = \dfrac{\sin^2\theta}{1-\cos\theta}$

If $\cos^2\theta + \sin^2\theta = 1$, Pythagorean Identity

then $\cos^2\theta = 1-\sin^2\theta$
and $\sin^2\theta = 1-\cos^2\theta$

I call these available substitutions "twisted pythagorous". They are useful!

Proof L.S. $= 1+\cos\theta$. RS $= \dfrac{(1-\cos^2\theta)}{1-\cos\theta}$, sub twisted pythag

$= \dfrac{(\cancel{1-\cos\theta})(1+\cos\theta)}{(\cancel{1-\cos\theta})}$, difference of squares factoring in numer.

$= (1+\cos\theta)$, divided common factor to 1.

$\because$ LS = RS

$\therefore$ Proven.

c) $\tan^2\theta + 1 = \dfrac{1}{\cos^2\theta}$

Proof

	L.S.	R.S.
Sub'n	$=\left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)+1$	$= \dfrac{1}{\cos^2\theta}$
Built a common denom'n	$= \dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta}$	
Add 'em'n	$= \dfrac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}$	
Pythag Identity Sub'n	$= \dfrac{(1)}{\cos^2\theta}$	

$\because$ LS = RS

$\therefore$ Proven

d) $(\sin x - \cos x)^2 = 1 - 2\sin x\cos x$

Proof

LS	RS
$= \sin^2 x - 2\sin x\cos x + \cos^2 x$	$= 1 - 2\sin x\cos x$
$= (\sin^2 x + \cos^2 x) - 2\sin x\cos x$	
$= (1) \quad - 2\sin x\cos x$	

Expanded the binomial using FOIL and collected like terms

Sub in using Pythagorean Identity

$\because$ LS = RS

$\therefore$ Proven

W.S. #1-23 odd numbers.

Proving Trig Identities Worksheet

Prove the following identities using proper form and showing all steps:

1. $\cot x = \csc x \cos x$
2. $\cot x \sin x = \cos x$
3. $\dfrac{\tan x}{\sec x} = \sin x$
4. $\tan x \cos x = \sin x$
5. $\dfrac{\cot x}{\csc x} = \cos x$
6. $\sin x \sec x = \tan x$
7. $\tan x \csc x = \sec x$
8. $\sec x(1 + \cos x) = 1 + \sec x$
9. $\sin x(1 + \csc x) = \sin x + 1$
10. $\tan x(1 + \cot x) = 1 + \tan x$
11. $\cos x(\sec x + 1) = \cos x + 1$
12. $\csc y(\sin y - 1) = 1 - \csc y$
13. $\cot z(1 - \tan z) = \cot z - 1$
14. $\sin y \tan^2 y \cot^3 y = \cos y$
15. $\sin^2 x \sec^2 x = \sec^2 x - 1$
16. $(1 + \tan^2 x)\cos^2 x = 1$
17. $(1 + \tan y)^2 - \sec^2 y = 2\tan y$
18. $(\cos x - \sin x)^2 = 1 - 2\sin x \cos x$
19. $(\sin y + \cos y)(\sin y - \cos y) = 1 - 2\cos^2 y$
20. $\dfrac{\tan^2 x - 1}{\cot^2 x - 1} = 1 - \sec^2 x$
21. $\dfrac{1}{1 + \tan^2 y} + \dfrac{1}{1 + \cot^2 y} = 1$
22. $\dfrac{\tan^2 z}{1 + \tan^2 z} = \sin^2 z$
23. $\dfrac{1 + \cos x}{1 - \cos x} = 1 + \dfrac{2\cos x(1 + \cos x)}{\sin^2 x}$
24. $\dfrac{\sec z}{\csc^2 z} = \sec z - \cos z$
25. $\csc^2 y - \csc y \cot y = \dfrac{1}{1 + \cos y}$
26. $\dfrac{1 + \sin^2 x \sec^2 x}{1 + \cos^2 x \csc^2 x} = \sin^2 x \sec^2 x$
27. $2 + \dfrac{\sin^4 y + \cos^4 y}{\sin^2 y \cos^2 y} = \sec^2 y \csc^2 y$
28. $(1 + \tan^2 z)\cos^2 z = 1$
29. $(1 - \cos^2 x)(1 + \tan^2 x) = \tan^2 x$
30. $\sin^2 x \sec^2 x = \sec^2 x - 1$
31. $\dfrac{\sin y + \cos y}{\sec y + \csc y} = \dfrac{\cos y}{\csc y}$
32. $\cos t + \sin t = \dfrac{1 + \tan t}{\sec t}$
33. $\dfrac{1}{1 - \sec x} + \dfrac{1}{1 + \sec x} = -2\cot^2 x$
34. $\dfrac{(\sin x + \cos x)^2}{(\sin x - \cos x)^2} = \dfrac{\sec^2 x + 2\tan x}{\sec^2 x - 2\tan x}$
35. $\dfrac{1 - \cos^2 x}{\sin x} = \sin x$
36. $1 + \dfrac{1}{\tan^2 x} = \dfrac{1}{\sin^2 x}$
37. $\dfrac{1}{\cos x} - \cos x = \sin x \tan x$
38. $\left(1 + \dfrac{1}{\tan^2 x}\right)\left(1 - \cos^2 x\right) = 1$
39. $\dfrac{1}{\cos^2 x} = 1 + \tan^2 x$
40. $\cos^2 x + \dfrac{\sin x \cos x}{\tan x} = 2\cos^2 x$
41. $\left(1 - \cos^2 x\right)\left(1 + \tan^2 x\right) = \tan^2 x$
42. $\dfrac{\sin x}{1 - \cos x} - \dfrac{1 + \cos x}{\sin x} = 0$