

Example 3 Simplifying Expressions

Expand and simplify $2(3x + 4)^2 - (4x + 5)(4x - 5)$.

Solution

$$\begin{aligned}2(3x + 4)^2 - (4x + 5)(4x - 5) &= 2(9x^2 + 24x + 16) - (16x^2 - 25) \\ &= 18x^2 + 48x + 32 - 16x^2 + 25 \\ &= 2x^2 + 48x + 57\end{aligned}$$

Key Concepts

1 To square a binomial, use one of the following patterns.

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

2 To find the product of the sum and difference of two terms, use the following pattern.

$$(a + b)(a - b) = a^2 - b^2$$

Communicate Your Understanding

1. Describe how you would expand and simplify each of the following.

a) $(2x + 1)^2$ **b)** $(3x - 4)^2$ **c)** $(4x - 5)(4x + 5)$

2. Using examples, explain why the product of the sum and difference of two terms has only two terms in the expansion.

Practice

A

1. What is the first term in each product?

a) $(x + 7)^2$ **b)** $(a - 9)^2$ **c)** $(2x - 1)^2$ **d)** $(9t + 5)^2$
e) $(3y + 6)^2$ **f)** $(7p - 2)^2$ **g)** $(4j + 1)^2$ **h)** $(6q - 8)^2$

2. What is the middle term, including its sign, in each product?

a) $(x - 3)^2$ **b)** $(y + 8)^2$ **c)** $(x + y)^2$ **d)** $(a - b)^2$
e) $(2x + 3)^2$ **f)** $(4a - 5)^2$ **g)** $(3x + 2y)^2$ **h)** $(6p - 7)^2$

3. Write the missing factor.

a) $(a - 7)(\square) = a^2 - 49$ **b)** $(x + 2)(\square) = x^2 - 4$
c) $(\square)(3m - 7) = 9m^2 - 49$ **d)** $(9x + 8)(\square) = 81x^2 - 64$
e) $(x - y)(\square) = x^2 - y^2$ **f)** $(\square)(2x + 3b) = (4a^2 - 9b^2)$

4. Expand.

a) $(x + 5)^2$ **b)** $(y + 1)^2$
c) $(x - 6)^2$ **d)** $(m - 3)^2$
e) $(x + 3)(x - 3)$ **f)** $(y + 6)(y - 6)$
g) $(m - 7)(m + 7)$ **h)** $(t - 8)(t + 8)$

5. Expand.

a) $(3x + 2)^2$

b) $(5x - 1)^2$

c) $(2x + 3)(2x - 3)$

d) $(2m + 7)^2$

e) $(3y - 2)(3y + 2)$

f) $(4y - 3)^2$

g) $(1 + 5m)(1 - 5m)$

h) $(2 - 3t)^2$

6. Expand.

a) $(2x + 3y)(2x - 3y)$

b) $(2x + 3y)^2$

c) $(3a - b)(3a + b)$

d) $(4t - 5s)^2$

e) $(4m - 5n)(4m + 5n)$

f) $(3c + 7d)^2$

g) $(y + 6x)(y - 6x)$

h) $(a - 8b)^2$

7. Expand and simplify.

a) $(x + 4)^2 + (x - 2)^2$

b) $(y + 6)(y - 6) + (y + 7)^2$

c) $(m - 8)^2 - (m - 1)(m + 1)$

d) $2(a + 3)(a - 3) + 3(a + 2)^2$

e) $4(2x + 1)^2 - 2(3x - 7)^2$

f) $5(3t - 1)^2 - 4(4t - 5)(4t + 5)$

8. Expand and simplify.

a) $(x - 7)(x + 5) - 2(x + 6)^2$

b) $(2x - 3)^2 - (3x - 1)(4x + 5)$

c) $3(3m + 1)(m - 4) - 4(2m + 3)(2m - 3)$

d) $3t^2 - (3 - 2t)^2 + 5(2t - 1)(2t + 1)$

e) $12 - 2(3y + 1)^2 - (y - 9)(3y + 2)$

f) $2(1 - 3t)^2 - 4(1 - 3t)(1 + 3t)$

g) $4(5s + t)(5s - t) - 6(3t^2 - t)$

h) $2(3m - n)^2 - 3(2m - 5)(m + 3)$

i) $(x + 2y)(x - 2y) + (2x + y)^2$

j) $3(a - 2b)^2 - 4(2a + b)^2$

Applications and Problem Solving

9. a) Verify that $(x + 3)^2 \neq x^2 + 9$ by substituting 1 for x .

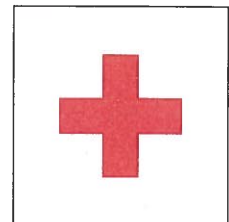
b) Expand $(x + 3)^2$ correctly.

10. **Flag areas** The International Red Cross, which is staffed mainly by volunteers, has twice won the Nobel Peace Prize for its medical and welfare work around the world. The flag of the International Red Cross is a square. For two International Red Cross flags of different sizes, the difference between the side lengths is 10 cm.

a) Let the shorter side length be x . Write an expression for the longer side length.

b) Write an expression that represents the area of each flag. Expand and simplify, if possible.

c) If x represents 25 cm, what is the area of each flag, in square centimetres?



B

11. Rewrite each multiplication as a product of the sum and difference of two numbers, as shown. Copy and complete the table.

Numbers	$(a + b)(a - b)$	Product
33×27	$(30 + 3)(30 - 3)$	891
24×16		
47×53		
62×58		

12. Multiply each of the following, using the method in question 11.

a) $(10 + 2)(10 - 2)$

b) $(15 + 3)(15 - 3)$

c) $(20 - 2)(20 + 2)$

d) 14×6

e) 17×23

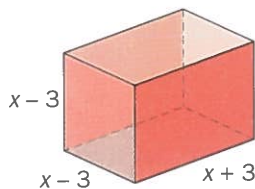
f) 32×28

13. Use $x = y - 2$ to write each of the following in terms of y . Then, expand and simplify.

a) $x^2 - 2x + 3$

b) $3x^2 + 5x - 9$

14. Measurement Write, expand, and simplify an expression that represents the surface area of the rectangular prism.



15. Measurement The length of an edge of a cube is represented by the expression $2x - y$. Write, expand, and simplify an expression that represents the surface area of the cube.

16. Rewrite in the form $(a + b)^2$ or $(a - b)^2$.

a) $x^2 + 14x + 49$

b) $x^2 - 16x + 64$

c) $4a^2 + 12a + 9$

d) $9b^2 - 24b + 16$

e) $64m^2 - 32m + 4$

f) $81n^2 + 90n + 25$

17. Change one term in each trinomial, so that it can be written as the square of a binomial.

a) $x^2 + 12x + 18$

b) $a^2 + 7a + 16$

c) $y^2 - 9y + 9$

d) $m^2 - 4m + 16$

e) $4x^2 - 4x + 2$

f) $9y^2 + 10y + 4$

18. Expand and simplify.

a) $(x^2 + 1)^2$

b) $(y^2 - 1)^2$

c) $(x^2 + y^2)^2$

d) $(x^2 - y^2)^2$

e) $(2x^2 + 3)^2$

f) $(3y^2 - 4)^2$

g) $(x^2 - 2y^2)^2$

h) $(4x^2 + 3y^2)^2$

19. Expand and simplify.

a) $(x^2 + 1)(x^2 - 1)$

b) $(y^2 - 2)(y^2 + 2)$

c) $(x^2 + y^2)(x^2 - y^2)$

d) $(8a^2 + 3)(8a^2 - 3)$

e) $(3x^2 + 2y^2)(3x^2 - 2y^2)$

f) $(4 - 3c^2)(4 + 3c^2)$

20. Measurement The side length of a square is represented by x centimetres. The length of a rectangle is 3 cm greater than the side length of the square. The width of the rectangle is 3 cm less than the side length of the square. Which figure has the greater area and by how much?

21. Communication If a square garden is made into a rectangle by shortening two opposite sides by 5 m each and lengthening the other two sides by 5 m each, how do the areas of the original garden and the new garden compare? Explain.

R.S. = 13 **b)** $x^2 + 8x + 12$ **7. a)** The length of the rectangle is $3x + 2y$. The width is $2x + y$. The area is $(3x + 2y)(2x + y)$. **b)** $6x^2 + 7xy + 2y^2$
8. a) $3x^2 + 13xy + 4y^2$ **b)** $8a^2 - 22ab + 5b^2$
c) $20m^2 - 7mn - 6n^2$ **d)** $20s^2 - 39st + 18t^2$
e) $7a^2 + ab - 8b^2$ **f)** $-6a^2 - ab + 12b^2$
9. a) $(x + 10)(x + 5)$ **b)** 750 m^2 **10. a)** $7x^2 - 72x + 20$
b) 29 280 cm^2 ; 2.928 m^2 **11. a)** $x^2 + x - 2$
b) $x^2 + 3xy + 2y^2 + 3x - 3y$ **12. a)** $14x^2 + 17x - 3$
13. a) 42 cm^2 **b)** $(n + 2)(n + 3)$ **c)** $n^2 + 5n + 6$
d) 930 cm^2 **14. a)** $10x^2 + 10x - 10$ **b)** 290 cm^2
15. No, the product of $(x - 1)$ and $(x + 1)$ is $x^2 - 1$, which is a binomial. The product of $(a + b)$ and $(c + d)$ is $ac + ad + bc + cd$, which has four terms. **16. a)** Take four consecutive numbers. Subtract the product of the outer numbers from the product of the inner numbers. **b)** 2, 2, 2, 2 **c)** $(x + 1)(x + 2) - (x)(x + 3) = 2$
d) The expression simplifies to 2.

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a) The product of three consecutive numbers plus the middle number. **b)** 8, 27, 64, 125 **c)** The answer is the cube of the middle number.
d) $(x - 1)(x + 1) + x = x^3$ **e)** The expression simplifies to x^3 .

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Practice 1. a) x^2 **b)** a^2 **c)** $4x^2$ **d)** $81t^2$ **e)** $9y^2$ **f)** $49p^2$
g) $16j^2$ **h)** $36q^2$ **2. a)** $-6x$ **b)** $+16y$ **c)** $+2xy$ **d)** $-2ab$
e) $+12x$ **f)** $-40a$ **g)** $+12xy$ **h)** $-84p$ **3. a)** $a + 7$ **b)** $x - 2$
c) $3m + 7$ **d)** $9x - 8$ **e)** $x + y$ **f)** $2a - 3b$
4. a) $x^2 + 10x + 25$ **b)** $y^2 + 2y + 1$ **c)** $x^2 - 12x + 36$
d) $m^2 - 6m + 9$ **e)** $x^2 - 9$ **f)** $y^2 - 36$ **g)** $m^2 - 49$ **h)** $t^2 - 64$
5. a) $9x^2 + 12x + 4$ **b)** $25x^2 - 10x + 1$ **c)** $4x^2 - 9$
d) $4m^2 + 28m + 49$ **e)** $9y^2 - 4$ **f)** $16y^2 - 24y + 9$
g) $1 - 25m^2$ **h)** $4 - 12t + 9t^2$ **6. a)** $4x^2 - 9y^2$
b) $4x^2 + 12xy + 9y^2$ **c)** $9a^2 - b^2$ **d)** $16t^2 - 40ts + 25s^2$
e) $16m^2 - 25n^2$ **f)** $9c^2 + 42cd + 49d^2$ **g)** $y^2 - 36x^2$
h) $a^2 - 16ab + 64b^2$ **7. a)** $2x^2 + 4x + 20$
b) $2y^2 + 14y + 13$ **c)** $-16m + 65$ **d)** $5a^2 + 12a - 6$
e) $-2x^2 + 100x - 94$ **f)** $-19t^2 - 30t + 105$
8. a) $-x^2 - 26x - 107$ **b)** $-8x^2 - 23x + 14$
c) $-7m^2 - 33m + 24$ **d)** $19t^2 + 12t - 14$
e) $-21y^2 + 13y + 28$ **f)** $54t^2 - 12t - 2$
g) $100s^2 - 22t^2 + 6t$ **h)** $12m^2 - 12mn + 2n^2 - 3m + 45$
i) $5x^2 + 4xy - 3y^2$ **j)** $-13a^2 - 28ab + 8b^2$

Applications and Problem Solving 9. a) L.S. = 16, R.S. = 10 **b)** $x^2 + 6x + 9$ **10. a)** $x + 10$ **b)** $x^2, (x + 10)^2 = x^2 + 20x + 100$ **c)** 625 cm^2 ; 1225 cm^2

11. $(20 + 4)(20 - 4) = 384$; $(50 - 3)(50 + 3) = 2491$;
 $(60 + 2)(60 - 2) = 3596$ **12. a)** 96 **b)** 216 **c)** 396 **d)** 84
e) 391 **f)** 896 **13. a)** $y^2 - 6y + 11$ **b)** $3y^2 - 7y - 7$
14. $2(x - 3)^2 + 4(x - 3)(x + 3) = 6x^2 - 12x - 18$
15. $6(2x - y)^2 = 24x^2 - 24xy + 6y^2$ **16. a)** $(x + 7)^2$
b) $(x - 8)^2$ **c)** $(2a + 3)^2$ **d)** $(3b - 4)^2$ **e)** $(8m - 2)^2$
f) $(9n + 5)^2$ **17. a)** $x^2 + 12x + 36$ **b)** $a^2 + 8a + 16$
c) $y^2 - 6y + 9$ **d)** $m^2 - 8m + 16$ **e)** $4x^2 - 4x + 1$
f) $9y^2 + 12y + 4$ **18. a)** $x^4 + 2x^2 + 1$ **b)** $y^4 - 2y^2 + 1$
c) $x^4 + 2x^2y^2 + y^4$ **d)** $x^4 - 2x^2y^2 + y^4$ **e)** $4x^4 + 12x^2 + 9$
f) $9y^4 - 24y^2 + 16$ **g)** $x^4 - 4x^2y^2 + 4y^4$
h) $16x^4 + 24x^2y^2 + 9y^4$ **19. a)** $x^4 - 1$ **b)** $y^4 - 4$ **c)** $x^4 - y^4$
d) $64a^4 - 9$ **e)** $9x^4 - 4y^4$ **f)** $16 - 9c^4$ **20.** The square, by 9 cm^2 . **21.** The original garden is 25 m^2 larger than the new garden. **22. a)** $a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
b) $4x^2 + 9y^2 + 1 + 12xy + 4x + 6y$ **23.** 1 term, 5 terms
24. a) {7, 24, 25}; {9, 40, 41}; {9, 12, 15}; {12, 35, 37},
{12, 16, 20}, {9, 12, 15}, {5, 12, 13} **b)** Let a represent the given leg, $c + b$ represent one factor of a^2 , and $c - b$ represent the other factor of a^2 . Thus $(c + b)(c - b) = a^2$, or $c^2 - b^2 = a^2$, or $c^2 = a^2 + b^2$. So $\{a, b, c\}$ is a Pythagorean triple, and represents the sides of a right triangle.

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1 Multiplying Binomials 1. a) $8x^2 + 42x + 27$
b) $6x^2 - 5x - 25$ **c)** $20y^2 - 52y + 33$
d) $32x^2 - 4xy - 21y^2$ **e)** $20x^2 + 7xy - 6y^2$ **f)** $9x - 14$
g) $7x^2 + 9x - 13$ **h)** $-6y^2 - 26y + 49$ **i)** $22x^2 + 12xy + y^2$
2 Special Products 1. a) $x^2 + 30x + 225$
b) $t^2 - 10t + 25$ **c)** $81 - 18y + y^2$ **d)** $16m^4 + 56m^2 + 49$
e) $36 - 60r + 25r^2$ **f)** $64x^2 + 48xy + 9y^2$
2. a) $4x^2 - 121$ **b)** $16 - 25x^2$ **c)** $9y^2 - 25x^2$ **3. a)** $4x$
b) $-4x - 13$ **c)** $44y^2 + 68y + 26$ **d)** $15 + 48m - 40m^2$

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Practice 1. a) $5(x + 5)$ **b)** not possible **c)** $9(y - 1)$
d) $3(x - 5y)$ **e)** $5x(5x + 2)$ **f)** $2a(2x + 4y - 3z)$
g) $pq(5r - s - 10t)$ **h)** $2(x^2 - x - 3)$ **i)** not possible
2. a) $9(a^3 + 3b^2)$ **b)** $3x(x^4 - 2x^2 + 3)$ **c)** $4y(3 - 2y + 6y^2)$
d) $6w^3(4w^2 + 1)$ **e)** not possible **f)** $11b(3a + 2c - b)$
g) $8xy(3y + 2x)$ **h)** $5y(7x - 2y)$ **i)** not possible
j) $12xy(2y - 1 + 3x)$ **k)** $9a^2b^2(3b + 1 - 2a)$
l) $6mn^2(m^2 + 3mn - 2)$ **3. a)** $(a + b)(5x + 3)$
b) $(x - 1)(3m + 5)$ not possible **d)** $(p + q)(4y - x)$
e) $(m + 7)(4t + 1)$ not possible **f)** not possible **4. a)** $(x + y)(w + z)$
b) $(x + 3)(y + 4)$ **c)** $(x + 1)(x - y)$ **d)** $(m + 4)(m - n)$
e) $(x + 2)(2x + 3y)$ **f)** $(t - 2)(5m^2 + t)$