2.4 *Evaluating Logs*

Warmup:

a) $3^3 \times 3^4 = 3^7$

b) $4^8 \div 4^4 = 4^4$

c) $5^9 \div 5^2 = 5^7$

d) $2^4 \times 2^2 = 2^6$

e) $(1^3)^4 = 1^{12}$

f) $(-7)^{2013} = -7^{2013}$

g) $\dfrac{(3^2)^3 \times 3}{(3^4)^2 \times 3^3}$
$= \dfrac{3^6 \times 3^1}{3^8 \times 3^3}$
$= 3^7 / 3^{11}$
$= 3^{-4}$

Recall: A logarithm is an exponent

Remark: Logs manipulate large (small) numbers to smaller (larger), more manageable numbers.

Ex Earthquake of magnitude 5 is $10^{5-3} = 100X$ more intense than a magnitude 3 quake.

Ex Interior pH of an old cheddar cheese for which $[H^+] = 0.0000062$: $pH = \log\left[\frac{1}{[H^+]}\right] = \log\left(\frac{1}{0.0000062}\right) \doteq 5.21$

Slightly acidic.

$y = \log_a x$ $\quad x = a^y$, by def'n.

exponent

base

Ex. Evaluate

a) $\log 100$
$= 2$

Why? Since $10^2 = 100$

b) $\log_2 16$
$= 4$

c) $\log_4 \frac{1}{2}$
$= -\frac{1}{2}$
$4^{-\frac{1}{2}} = \frac{1}{2}$

d) $\log_{\frac{1}{2}} 4$
$= -2$

Why? Since $\left(\frac{1}{2}\right)^{-2} = \left(\frac{2}{1}\right)^2 = 4$

Properties of Logs If $a>0, a \neq 1$ then:

a) $\log_a a^x = x$ b) $a^{\log_a x} = x$ c) $\log_a 1 = 0$

Inverse Property

Check: $a^0 = 1$ ✓

Equivalent

Proof: Sub $y = \log_a x$ into $a^y = x$

$a^{\log_a x} = x$

Evaluate

a) $\log_2 x$ if $x = 2, 4, 8, 16, 32$ — Power Sequence Base 2

$y = 1, 2, 3, 4, 5$

b) $\log_{10} x$ if $x = \frac{1}{10}, 1, 10, 100, 1000$ — Power Sequence Base 10.

$y = -1, 0, 1, 2, 3$

Evaluate

a) $\log_3 3^{10}$

asks $3^? = 3^{10}$

$\therefore\ ? = 10$

b) $\log_{27} 3^7$ — get same bases

$= \log_{27} (27^{\frac{1}{3}})^7$, sub

$= \log_{27} (27^{\frac{7}{3}})$

$= 7/3$

c) $\log_{10} 1$

$= 0$

d) $7^{\log_7 2}$

$= 2$, inverse property

e) $2^{\log_8 27}$

$= (2^{\frac{3}{3}})^{\log_8 27}$, sub ← get same bases

$= ((2^{\frac{3}{1}})^{\frac{1}{3}})^{\log_8 27}$

$= 8^{\frac{1}{3}\log_8 27}$

$= (8^{\log_8 27})^{\frac{1}{3}}$

$= (27)^{\frac{1}{3}}$

$= 3$

→ Sol'n: Replace 2 with 8 to build same bases, then make 8 = 2 using exponent laws. So,

$2^{\log_8 27}$

$= (8^{\frac{1}{3}})^{\log_8 27}$, sub

$= (8^{\log_8 27})^{\frac{1}{3}}$, switch exponents

$= (27)^{\frac{1}{3}}$, inverse property

$= 3$, cube root

f) $7^{\log_{\sqrt{7}} 2}$ ← Manipulate the base of the power to equal base of the log

$= ((7^{\frac{1}{2}})^{\frac{2}{1}})^{\log_{\sqrt{7}} 2}$

$= (\sqrt{7}^{\log_{\sqrt{7}} 2})^2$

$= (2)^2$

$= 4$

, replaced 7 with $7^{\frac{1}{2}}$ and then made $7^{\frac{1}{2}} = 7$ by squaring

$y = \log_a x \iff x = a^y$, by def'n.

exponent

base

g) $(8)^{\log_{512} 27}$

Sol'n 512 is a power of 8: $8^3 = 512$

So $= (512^{\frac{1}{3}})^{\log_{512} 27}$, get same bases

$= (512^{\log_{512} 27})^{\frac{1}{3}}$, switch exponents

$= (27)^{\frac{1}{3}}$, inverse property

$= 3.$ ✓

W.S. #1, 2

P117 #(1-4, 7) every other letter