

# 2.4 \* Evaluating Logs \*

Warmup:

a)  $3^3 \times 3^4 = 3^7$   
 b)  $4^8 \div 4^4 = 4^4$   
 c)  $5^9 \div 5^2 = 5^7$   
 d)  $2^9 \times 2^2 = 2^6$

e)  $(1^3)^4 = 1^{12}$   
 f)  $(-7)^{2013} = -7^{2013}$   
 g)  $\frac{(3^3)^3 \times 3}{(3^4)^2 \times 3^3}$   
 $= \frac{3^6 \times 3^1}{3^8 \times 3^3}$   
 $= 3/3^{11}$   
 $= 3^{-4}$

Recall: A logarithm is an exponent (small) (larger)

Remark: Logs manipulate large numbers to smaller, more manageable numbers.

Ex Earthquake of magnitude 5 is  $10^{5-3} = 100$  X more intense than a magnitude 3 quake.

Ex Interior pH of an old cheddar cheese for which  $[H^+] = 0.0000062$   $\times$   $pH = \log \left[ \frac{1}{[H^+]} \right] = \log \left( \frac{1}{0.0000062} \right) \approx 5.21$   $\uparrow$

Slightly acidic.

$y = \log_a x$



$x = a^y$ , by def'n.

Ex. Evaluate

a)  $\log 100 = 2$   
 b)  $\log_2 16 = 4$   
 c)  $\log_4 \frac{1}{2} = -\frac{1}{2}$   
 d)  $\log_{\frac{1}{2}} 4 = -2$

Why? Since  $10^2 = 100$

Why? Since  $(\frac{1}{2})^{-2} = (\frac{2}{1})^2 = 4$

Properties of Logs If  $a > 0, a \neq 1$  then:

a)  $\log_a a^x = x$  b)  $a^{\log_a x} = x$  c)  $\log_a 1 = 0$

Inverse Property

check:  $a^0 = 1$  ✓

Evaluate

a)  $\log_2 x$  if  $x = 2, 4, 8, 16, 32$   
 $y = 1, 2, 3, 4, 5$

Power Sequence Base 2

b)  $\log_{10} x$  if  $x = \frac{1}{10}, 1, 10, 100, 1000$   
 $y = -1, 0, 1, 2, 3$

Power Sequence Base 10.

Proof: Sub  $y = \log_a x$  into  $a^y = x$   
 $a^{\log_a x} = x$

Evaluate

a)  $\log_3 3^9$

b)  $\log_{27} 3^7$

c)  $\log_{10} 1 = 0$

d)  $7^{-\frac{1}{2}}$

get same bases

asks  $3^3 = 3^9$   
 $\therefore ? = 10$

$= \log_{27} (27^{\frac{7}{3}})$ , sub  
 $= \log_{27} (27^{\frac{7}{3}})$   
 $= \frac{7}{3}$

$= -2$ , inverse property

e)  $2^{\log_8 27}$

$= (2^{\frac{2}{3}})^{\log_8 27}$ , sub  
 $= ((2^{\frac{2}{3}})^{\frac{1}{3}})^{\log_8 27}$   
 $= 2^{\frac{2}{9} \log_8 27}$   
 $= (8^{\log_8 27})^{\frac{2}{9}}$   
 $= (27)^{\frac{2}{9}}$   
 $= 3$

Soln: Replace 2 with 8 to build same bases, then make  $8 = 2$  using exponent laws. So,  
 $\log_8 27$   
 $= (8^{\frac{1}{3}})^{\log_8 27}$ , sub  
 $= (2^{\log_8 27})^{\frac{1}{3}}$ , switch exponents  
 $= (27)^{\frac{1}{3}}$ , inverse property  
 $= 3$ , cube root

f)  $7^{\frac{1}{2}}$

$= (7^{\frac{1}{4}})^{\log_{\sqrt{7}} 2}$   
 $= (\sqrt{7})^{\log_{\sqrt{7}} 2}$   
 $= (2^{\frac{1}{2}})^{\log_{\sqrt{7}} 2}$   
 $= 2$

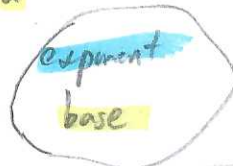
Manipulate the base of the power to equal base of the log, replaced 7 with  $7^{\frac{1}{2}}$  and then made  $7^{\frac{1}{2}} = 7$  by squaring

g)  $(8)^{\log_{512} 27}$

Soln 512 is a power of 8:  $8^3 = 512$

So  $= (512^{\frac{1}{3}})^{\log_{512} 27}$ , get same bases  
 $= (512^{\log_{512} 27})^{\frac{1}{3}}$ , switch exponents  
 $= (27)^{\frac{1}{3}}$ , inverse property  
 $= 3$  ✓

$y = \log_a x \iff x = a^y$ , by def'n.



W.S. # 1, 2

NPII7 # (1-4, 7) every other letter