$N_{2.5}$ * 3 Laws of Logs *

Warmup: 1. Solve for x.

a) $\log_4 16^{-1} = x$

Sol'n By def'n, $4^x = 16^{-1}$

$4^x = (4^2)^{-1}$, same bases

$4^x = 4^{-2}$, like powers

So $x = -2$ ✓, logic

✓ $x = a^y$

✓ $y = \log_a x$

b) $\log_{(x-1)}(7-x) = 2$

Sol'n $(x-1)^2 = 7-x$, by def'n.

$x^2 - 2x + 1 = 7 - x$, expanded.

$x^2 - 1x - 6 = 0$, quadratic eq'n.

$(x-3)(x+2) = 0$, sum + product

$\therefore x = 3$ or $x = -2$.

But $y = \log_a x$ has 3 restrictions aka conditions: ① $a \neq 1$, ② $a > 0$, and ③ $x > 0$

So for warmup b) ① $x - 1 \neq 1$, $x \neq 2$ and ② $x - 1 > 0$, $x > 1$ and ③ $7 - x > 0$, $7 > x$ $\therefore$ $\underline{x = 3}$ ✓✓ ($x = -2$ inadmissible)

Remark: If $a > 0$, $M > 0$ and $N > 0$, $n \in \mathbb{R}$, then $\boxed{\log_a MN = \log_a M + \log_a N}$

Product law

That is: "The log of a product equals the sum of the logs of the factors."

Proof Let $\log_a M = x \Rightarrow a^x = M$, definition of log

Let $\log_a N = y \Rightarrow a^y = N$, definition of log

Then, $\log_a MN = \log_a (a^x a^y)$, substitution

$= \log_a (a^{x+y})$, like powers. Add exponents

$= x + y$

$\therefore \log_a MN = \log_a M + \log_a N$, substitution

Ex: Simplify. $\log_2 (32 \times 64)$

Sol'n

$= \log_2 32 + \log_2 64$, Product Law.

$= 5 + 6$

$= 11$ ✓

e)

Remark$_2$: $\boxed{\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N}$. That is, "The log of a quotient is equal to the log of the numer minus log of denom."

Quotient Law

Proof Let $\log_a M = x \rightarrow a^x = M$, by def'n
$\log_a N = y \rightarrow a^y = N$, by def'n.

Then $\log_a\left(\frac{M}{N}\right) = \log_a\left(\frac{a^x}{a^y}\right)$, substitution
$= \log_a\left(a^{x-y}\right)$, like powers
$= x - y$, evaluate the log.
$\therefore \log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$, substitution

Ex Find the value of
$\log_2 144 - \log_2 9$

Sol'n "Appears impossible to evaluate since 144 is not a power of 2." But using Quotient Law $\div$ $= \log_2\left(\frac{144}{9}\right)$
$= \log_2(16)$
$= 4$ ✓

Remark$_3$: $\boxed{\log_a (M)^n = n\log_a M}$ That is, "The log of a power is equal to the exponent multiplied to the log of the base."

Power Law

Proof Let $a^n = M$ and Let $a^n = M$
Then, $a^{np} = M^p$ Then, $\log_a a^n = \log_a M$
$\log_a a^{np} = \log_a M^p$ ② $n = \log_a M$
$pn = \log_a M^p$ ①

Sub ② into ①
$\therefore p\log_a M = \log_a M^p$ DONE ✓

Ex Simplify. $\log_3 9^4$

Sol'n $= (4)(\log_3 9)$, "drop down 4"
$= (4)(2)$, evaluate log
$= 8$, tidy up.

Ex$_2$ Simplify. (Hint: Change roots to fractions)
$\log_3 \sqrt[4]{3} + \log_3 \sqrt[5]{81}$

Sol'n$_1$ "Sum of like logs = log of product of big numbers."
$= \log_3 3^{1/4} + \log_3 81^{\frac{1}{5}}$
$= \log_3\left(3^{1/4} \times 81^{1/5}\right)$, Product Law
$= \log_3\left(3^{1/4} \times 3^{4/5}\right)$, since $81 = 3^4$ and $(3^4)^{\frac{1}{5}} = 3^{4/5}$
$= \log_3\left(3^{\frac{5+16}{20}}\right) = \frac{21}{20}$, evaluate the log.

Sol'n$_2$ "Much easier if you see it"
$= \log_3 3^{1/4} + \log_3 81^{1/5}$
$= \left(\frac{1}{4}\right)\log_3 3 + \left(\frac{1}{5}\right)\log_3 81$
(Power Law 2X)
$= \left(\frac{1}{4}\right)(1) + \left(\frac{1}{5}\right)(4)$
$= \frac{5}{20} + \frac{16}{20}$, built common denom
$= \frac{21}{20}$

Ex$_3$ Simplify. $5^{(\log_5 8 - \log_5 2)}$

Sol'n >> Apply the 3 laws of logs as much as possible.

$= 5^{(\log_5(\frac{8}{2}))}$, Quotient Law

$= 5^{\log_5(4)}$

$= 4$, inverse property: $a^{\log_a x} = x$.

Done.

Ex$_4$ If $\log_3 x = 0.2$, find $\log_3 (x\sqrt{x})$ ②

Sol'n$_1$ $3^{0.2} = x$, by def'n

① $3^{\frac{1}{5}} = x$

Sub ① → ②

"Changed root to exponent"

$\log_3 [3^{\frac{1}{5}} \sqrt{3^{1/5}}]$

$= \log_3 [3^{\frac{1}{5}} 3^{\frac{1}{10}}]$, $(3^{\frac{1}{5}})^{\frac{1}{2}} = 3^{\frac{1}{10}}$

$= \log_3 [3^{\frac{2}{10} + \frac{1}{10}}]$, like powers

$= \log_3 3^{3/10}$

$= \frac{3}{10}$, evaluated log.

✓

Sol'n$_2$ "If you can see it"

$\log_3 x\sqrt{x}$

$= \log_3 x + \log_3 \sqrt{x}$, product law

$= \log_3 3^{\frac{1}{5}} + \log_3 (3^{\frac{1}{5}})^{\frac{1}{2}}$, since $x = 3^{1/5}$ given

$= \frac{1}{5} + \log_3 3^{\frac{1}{10}}$, evaluate log

$= \frac{1}{5} + \frac{1}{10}$, evaluate log

$= \frac{3}{10}$, common denom

✓

P125 #(3-8) every other letter