

Key Concepts

The following formulas can be used to determine characteristics of geometric figures and to verify geometric properties.

$$\text{Slope of a line segment: } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Length of a line segment: } l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Midpoint of a line segment: } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{Point-slope form of the equation of a line: } y - y_1 = m(x - x_1)$$

$$\text{Slope and } y\text{-intercept form of the equation of a line: } y = mx + b$$

Communicate Your Understanding

- $\triangle DEF$ has vertices $D(5, 2)$, $E(0, 3)$, and $F(4, -3)$. Describe how you would verify that $\triangle DEF$ is an isosceles right triangle.
- $\triangle KLM$ has vertices $K(2, 2)$, $L(0, -2)$, and $M(4, 0)$. Describe how you would find an equation for each of the following. Explain and justify your reasoning.
 - the median from M to KL
 - the altitude from K to LM
 - the right bisector of KM
- The equation of a circle with centre $O(0, 0)$ is $x^2 + y^2 = 20$. The points $A(4, 2)$ and $B(-2, 4)$ are the endpoints of chord AB . Describe how you would verify that the centre of the circle lies on the right bisector of chord AB .

Practice

A

- A quadrilateral has vertices $O(0, 0)$, $P(3, 5)$, $Q(8, 6)$, and $R(5, 1)$. Verify that $OPQR$ is a parallelogram.
- Given the points $X(1, 4)$, $Y(-2, 2)$, and $Z(3, 1)$, verify that $\triangle XYZ$ is a right triangle.
- The vertices of a triangle are $K(2, 6)$, $L(4, 10)$, and $M(8, -2)$. Let P be the midpoint of KL and Q be the midpoint of LM . Verify that
 - PQ is parallel to KM
 - PQ is half the length of KM
- Verify that the quadrilateral with vertices $P(2, 3)$, $Q(5, -1)$, $R(10, -1)$, and $S(7, 3)$ is a rhombus.
- The vertices of a quadrilateral are $K(-1, 0)$, $L(1, -2)$, $M(4, 1)$, and $N(2, 3)$. Verify that
 - $KLMN$ is a rectangle
 - the lengths of the diagonals of $KLMN$ are equal

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1 Length of a Line Segment **1.** names the program; clears the memory; prompts user for x_1 -data and reads it into variable P; prompts user for y_1 -data and reads it into variable Q; prompts user for x_2 -data and reads it into variable R; prompts user for y_2 -data and reads it into variable S; assigns the difference $R - P$ to the variable X; assigns the difference $S - Q$ to the variable Y; assigns the square root of the sum of the squares of X and Y to the variable L; prints "LENGTH IS" and prints the value of L. **2. a)** 6.4 **b)** 17.1 **c)** 27.33

2 Midpoint of a Line Segment **1.** Change line 6 to $(R + P)/2 \rightarrow X$. Change line 7 to $(S + Q)/2 \rightarrow Y$. Delete line 8. Change the last line to $\text{DISP" MIDPOINT IS"}$. Add lines $\text{DISP"X="}, X$ and $\text{DISP"Y="}, Y$. **2. a)** $(-1, 6)$ **b)** $(2.5, -3.5)$ **c)** $(0.3, -3.45)$

3 Collinear Points **2. a)** collinear **b)** not collinear
3. Answers may vary. **a)** $(7, 6)$ **b)** $(-7, 6)$

Review: Equations of Lines pp. 85–87

1 Using the Point-Slope Form

1. a) $2x - y - 1 = 0$ **b)** $5x - y - 18 = 0$
c) $3x + y + 12 = 0$ **d)** $4x + y + 9 = 0$ **e)** $x - 2y - 18 = 0$
f) $x + 2y + 5 = 0$ **g)** $2x - 2y - 9 = 0$ **h)** $3x - 2y + 7 = 0$
i) $3x + 2y + 5 = 0$ **2. a)** $2x + y - 13 = 0$
b) $3x + y - 3 = 0$ **c)** $x - 2y - 9 = 0$ **d)** $2x - 5y - 11 = 0$
e) $5x - 4y + 22 = 0$ **f)** $4x - 3y + 12 = 0$ **3. a)** $y - 5 = 0$;
 $x - 4 = 0$ **b)** $y - 2 = 0$; $x + 3 = 0$ **c)** $y + 6 = 0$; $x + 5 = 0$
d) $y - 8 = 0$; $2x - 1 = 0$ **e)** $3y + 1 = 0$; $x - 9 = 0$
f) $y + 9 = 0$; $x = 0$ **g)** $y = 0$; $x + 1 = 0$ **h)** $y = 0$; $x = 0$

2 Using the Slope and y-Intercept Form

1. a) $3, 4$ **b)** $-4, 6$ **c)** $1, 5$ **d)** $\frac{1}{2}, -4$ **e)** $-\frac{1}{2}, \frac{4}{3}$ **f)** $\frac{1}{4}, -1$
g) $\frac{5}{2}, -3$ **h)** $-\frac{2}{3}, 0$ **i)** $-20, 6$ **2. a)** $1, 2$ **b)** $-1, 3$ **c)** $2, 5$
d) $-3, 10$ **e)** $\frac{1}{2}, -1$ **f)** $-\frac{1}{2}, 1$ **g)** $4, 7$ **h)** $-\frac{1}{5}, \frac{13}{5}$ **i)** $\frac{1}{3}, \frac{5}{3}$

3 Parallel and Perpendicular Lines

1. a) $3x - y - 5 = 0$ **b)** $2x - y + 8 = 0$ **c)** $x + 2y + 1 = 0$
d) $x - 3y + 24 = 0$ **2. a)** $x - 2y + 8 = 0$ **b)** $3x + y - 2 = 0$
c) $2x - y + 2 = 0$ **d)** $x + 2y - 5 = 0$

Section 2.4 pp. 95–99

Practice **1.** PQ and OR have slope $\frac{1}{5}$; OP and RQ have slope $\frac{5}{3}$ **2.** XY has slope $\frac{2}{3}$; XZ has slope $-\frac{3}{2}$; the slopes are negative reciprocals, so XY is perpendicular to XZ **3. a)** Both segments have

slope $-\frac{4}{3}$ and so are parallel. **b)** $PQ = 5$, $KM = 10$;
 $PQ = \frac{1}{2}KM$ **4.** Opposite sides are parallel (two have

slope 0 and two have slope $-\frac{4}{3}$) and all sides have length 5; so PQRS is a rhombus. **5. a)** KL and NM have slope -1 ; KN and LM have slope 1. Thus, opposite sides are parallel and adjacent sides are perpendicular. KLMN is a rectangle. **b)** KM and LN both have length $\sqrt{26}$ **6.** $x + y - 5 = 0$ **7. a)** The midpoint of BC is $(4, 3)$. The line containing $(4, 3)$ and $A(3, 4)$ has slope -1 . BC has slope 1. Thus, $A(3, 4)$ is on the perpendicular bisector of BC.

b) The midpoint of QR is $(0, 0)$. The line containing $(0, 0)$ and $P(1, 3)$ has slope 3. QR has slope $-\frac{1}{3}$. Thus, $P(1, 3)$ is on the perpendicular bisector of QR. **c)** The midpoint of LM is $(2, -2)$. The line containing $(2, -2)$ and $K(-2, -4)$ has slope $\frac{1}{2}$. LM has slope -2 . Thus, $K(-2, -4)$ is on the perpendicular bisector of LM. **8.** The midpoint of CD is $(2, -1)$.

The line containing $(2, -1)$ and $O(0, 0)$ has slope $-\frac{1}{2}$. CD has slope 2. Thus, $O(0, 0)$ is on the right bisector of CD. **9.** Slope AC is $\frac{1}{5}$ and slope BD is -5 . The slopes are negative reciprocals, so the diagonals AC and BD are perpendicular.

10. The opposite sides PQ and OR both have slope $\frac{1}{5}$ and so are parallel; the other two sides have different slopes, so are not parallel. OPRQ is a trapezoid.

11. PS and QR both have slope $\frac{2}{3}$; RS and QP are both vertical. Thus, opposite sides are parallel and PQRS is a parallelogram. **12. a)** KL has slope 1, KM has slope -1 . KL and KM are perpendicular. **b)** The midpoint of LM is a distance of $\sqrt{17}$ from each vertex. **13.** Opposite sides of the quadrilateral are parallel (two sides have slope $\frac{2}{5}$ and two have slope -5) and so the quadrilateral is a parallelogram.

14. a) $7x + 2y + 1 = 0$ **b)** $x - y + 1 = 0$ **c)** $x + 4y - 2 = 0$
15. a) $y = 0$, $x - 4 = 0$, $x + y - 4 = 0$ **b)** All three medians intersect at $(4, 0)$. **16.** $(4, 0)$ **17.** $(2, 2)$

Applications and Problem Solving **18.** The vertices of the triangle are $A(2, 0)$, $B(-4, 3)$, and $C(-1, -6)$.
 $AB = AC = \sqrt{45}$; AC has slope 2 and AB has slope $-\frac{1}{2}$