N 2.5 *Solving Exponential Equations*

Warmup: 1. Solve $9.1^x = 17$

Soln: ∵ 17 is not a power of 9.1

∴ We could guess and check approximate answers for x.

OR

We could use logs to speed up our efficiency.

Equivalently, $\underbrace{\log_{9.1}}17 = x$, definition of a log.

This is not useful because $\boxed{\log}$ is understood by calculator to be the base 10 log: $\boxed{\log_{10}}$

So apply $\log_{10} = \log$ to both sides of given equation.

$\log_{10} 9.1^x = \log_{10} 17$, kinda like squaring both sides of an equation. Recall, logs are exponents.

Cool Move: $x \log_{10} 9.1 = \log_{10} 17$, power law

$$\frac{x \cancel{\log_{10} 9.1}}{\cancel{\log_{10} 9.1}} = \frac{\log_{10} 17}{\log_{10} 9.1}$$

$x \doteq 1.28$, used calculator.

DONE.

In general: If $y = a^x$, then $\log y = \log a^x$

$\log y = x \log a$, power law

$\frac{\log y}{\log a} = x$

AND If $y = a^x$, then $\log_a y = x$, by def'n

$\therefore x = x$

$\therefore \boxed{\log_a y = \frac{\log y}{\log a}}$ Change of Base Formula

Yay! You are free to use your calculator to work with any and all log bases!

"$\log_a y$ equals log of big number y divided by log of the base a"

Try some: i) $\log_{(\frac{1}{4})}(2)$ ii) $\log_{(\sqrt{3})}(9)$

i) $= \frac{\log 2}{\log(\frac{1}{4})}$

$= -\frac{1}{2}$, calculator

ii) $= \frac{\log 9}{\log \sqrt{3}}$

$= 4$, calculator

Now for the lesson →

Ex1 Solve $3^x = 27$

Sol'n Strategy #1: "Like powers on both sides." (No logs used.)

$3^x = (3^3)$, substitution to get bases equal

$\therefore x = 3$, set exponents equal.

Ex2 Solve $2 = 20(0.4)^{\left(\frac{t}{7}\right)}$ to two decimal places.

Sol'n

$\frac{2}{20} = 0.4^{\frac{t}{7}}$

$0.1 = 0.4^{\frac{t}{7}}$

$\log 0.1 = \log 0.4^{\frac{t}{7}}$, log both sides

$\log 0.1 = \frac{t}{7} \log 0.4$, Strategy #2: "Bring down exponent law." Power Law.

$\frac{\log 0.1}{\log 0.4} = \frac{t}{7}$, divided

$\frac{7 \log 0.1}{\log 0.4} = t$, multiplied both sides by 7. → $t \doteq 17.59$, calculator

exact

approx

Ex3 Solve $-6^{(4x-5)} = -20$ to one decimal place.

Sol'n

$6^{(4x-5)} = 20$, mult by -1.

$\log 6^{(4x-5)} = \log 20$, log both sides

$(4x-5)\log 6 = \log 20$, power law

$4x - 5 = \frac{\log 20}{\log 6}$

$4x - 5 \doteq 1.6720$

$4x \doteq 6.6720$

$x \doteq 1.7$ ✓

Ex$_4$ Solve $\log_3 x = \frac{1}{2}\log_3 36 + \log_3 18 - \log_3 2$

Soln, Strategy #3: "Use laws of logs."

$\log_3 x = \log_3 36^{\frac{1}{2}} + \log_3 18 - \log_3 2$, power law: Raise exponent up onto big number 36.

$\log_3 x = \log_3 6 + \log_3 18 - \log_3 2$, simplify

$\log_3 x = \log_3 \left(\frac{6\times 18}{2}\right)$, Like Logs: Used → Product law, → Quotient law "Now have single log on both sides."

$\log_3 x = \log_3 54$, simplify big number

$\therefore\ x = 54$, logic

Note$_2$: $(\log_3 27)^{\frac{1}{2}} \neq \log_3 27^{\frac{1}{2}}$ and $\log_3 27^{\frac{1}{2}} = \frac{1}{2}\log_3 27$

Note$_3$: If $y = 3^x$

then ① $x = 3^y$, for inverse

$\therefore$ ② $\log_3 x = y$, definition of a log

$\therefore\ x = 3^{\log_3 x}$, subbed ② → ①

$\therefore\ x = x$, inverse property RHS

Just "playing with patterns."

N P126 #9,10

N P132 #(1-6) even.