

# N<sub>2.3</sub> \* Exponential Growth and Decay \* ↳ "A grade 11 U lesson"

Warmup: 1. If  $\log_9 4 = a$  and  $\log_{27} 5 = b$ , find an expression in terms of  $a$  and  $b$  for  $\log_3 20$ .

Soln: If  $\log_9 4 = a$ ; If  $\log_{27} 5 = b$ , Since  $\log_3 20$   
 then,  $9^a = 4$  then,  $27^b = 5$   
 $(3^2)^a = 4$   $(3^3)^b = 5$   
 $3^{2a} = 4$   $3^{3b} = 5$   
 $= \log_3 (4 \times 5)$   
 $= \log_3 [3^{2a} \times 3^{3b}]$   
 $= \log_3 (3^{2a+3b})$   
 $= 2a+3b$ , evaluated log

2. Find the total price, including 13% hst tax, on a \$5000 car price.

Soln: First change the tax aka growth rate aka percentage increase aka rate of change to a decimal and add one.

$$\begin{aligned} \text{Total Price} &= 5000(0.13+1) \\ &= 5000(1.13) \\ &= 5,650. \end{aligned}$$

Note:  $y = c(a)^x$  models exponential growth and decay.

From 3U  
 Future Amount  $\uparrow$  Initial Amount  $\uparrow$  Growth/Decay Factor  $\uparrow$  Time Periods.  
 $a > 1$  models growth  
 $a < 1$  models decay

Recall:  $a = 1+r$ ,  $x = \frac{t}{d} \leftarrow \text{total time}$ ,  $x = \frac{t}{h} \leftarrow \text{total time}$   
 $a = 1-r$ ,  $d \leftarrow \text{doubling time}$ ,  $h \leftarrow \text{half life time}$  match units

Ex: Your grandparents bought their house in 1966 for \$30,000. If the value increased 5% per year

a) Estimate house value in 2020.

Soln:  $y = c(a)^x$   
 $y = ?$   
 $c = 30000$   
 $1+r \rightarrow a = 1.05$   
 $x = 2020 - 1966 = 54$   
 $y = 30000(1.05)^{54}$   
 $y = 418,160.88$   
 $\therefore$  The house has a value of \$418,160.88.

b) When will the house be quintuple its 1966 value?

Soln

$$y = c(a)^x$$

$$\frac{(30000 \times 5)}{30000} = \frac{30000(1+0.05)^x}{30000}$$

$$5 = (1.05)^x$$

Now,  $\log 5 = \log 1.05^x$

$$\frac{\log 5}{\log 1.05} = \frac{x \log 1.05}{\log 1.05}, \text{ power law}$$

$$32.99 = x$$

∴ In 33 years the home will quintuple.

Ex<sub>2</sub> A car sells for \$20000. Each year it depreciates by 15%. What is the car's value after 6 years?  
↳ R.O.C.

Soln

$$y = c(a)^x = 20000(0.85)^6 = \underline{\underline{7,542.99}}$$

∴ In 6 years the car is worth \$7,542.99.

$$y = ?$$

$$c = 20000$$

$a = 1 - 0.15$  } Decay Factor "a"  
= 0.85 } Convert decay percent to a decimal and minus from 1.

$$x = 6.$$

Ex<sub>3</sub> A product of a nuclear explosion is plutonium-239, which has a half life of 24,000 years. What percentage is remaining after 1000 years?  $\left(\frac{1000}{24000}\right)$

Soln

time	% remaining
0	100
24000	50, 1 half life
48000	25, 2 half lives
72000	12.5, 3 half lives

$$y = ?$$

$$c = 100 \text{ for } 100\%$$

$$a = 1 - \frac{1}{2} = 0.50$$

$$y = 100(0.5)^{\left(\frac{1000}{24000}\right)}$$

$$y = \underline{\underline{97.15\%}}$$

$\frac{t}{h} \rightarrow x = \frac{1000}{24000}$ , convert given time,  $t$ , to part of 1 half life aka decay period. Match units

Ex<sub>4</sub> A population is growing exponentially. At time 0, the population is 35000. In 10 years the population is 44,400. Find the rate of growth "a".

Soln

$$y = c(a)^x$$

$$44400 = 35000(a)^{10}$$

$$1.269 = a^{10}$$

$$1.269^{\frac{1}{10}} = a$$

$$\sqrt[10]{1.269} = a$$

$$1.0241 = a$$

∴ So growth rate is 2.41% per year.

N P 110 # 2, 3, 6, 7, 11-13