Factoring and Expanding Review

Warmup: Remark₁: To simplify a rational expression

Step 1. Factor numerator
Step 2. Factor denominator. } "With an eye to "remove" a common factor or two."
Step 3. Divide common factor to one

These are great applications of factoring. You will study them in grade 11 too

Ex₁ Simplify

a) $\frac{12x^2}{15x}$

Sol'n $= \frac{3x(4x)}{3x(5)}$ ← factored numer ← factored denom

$= \frac{4x}{5}$, tidy up

b) $-\frac{6m^2n^3}{24mn^5}$

Sol'n $= \frac{6mn^3(-1m)}{6mn^3(4n^2)}$

$= \frac{-1m}{4n^2}$, simplified

c) $\frac{x^2-5x-6}{x^2-36}$

Sol'n $= \frac{(x-6)(x+1)}{(x-6)(x+6)}$ ← factor ← factor

$= \frac{x+1}{x+6}$, divided.

d) $\frac{4a^2-10}{a-3b} \times \frac{2a^2-18b^2}{6a^2-15}$

Sol'n $= \frac{2(2a^2-5)}{a-3b} \times \frac{2(a^2-9b^2)}{3(2a^2-5)}$ "factor-factor, divide common factor"

$= \frac{2}{a-3b} \times \frac{2(a-3b)(a+3b)}{3}$ "factor"

$= \frac{4(a+3b)}{3}$, divided common factor

e) $\frac{x^2+x-2}{x^2-x}$

Sol'n $= \frac{(x+2)(x-1)}{x(x-1)}$, factor, factor

$= \frac{x+2}{x}$, divide

Remark₂: You should know and apply Toolkit #3 list. Let's try a few examples.
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Ex₁ Factor.

a) $3ax-4x+6ay-8y$

Sol'n $= x(3a-4)+2y(3a-4)$

$= (3a-4)(x+2y)$

Group Factoring

b) $n^2-3n-40$

Sol'n $= (n-8)(n+5)$

Sum and Product Factoring

c) $12x^2-15x-18$

Sol'n $= 3(4x^2-5x-6)$, Common Factor

$= 3(4x+3)(x-2)$, "Tricky Tri" ↳

	a	c	
2	4	3	-3
2	1	-2	2

$\underline{Ex_2}$ Expand and simplify

a) $(3x-y)^2$

Soln $= (3x-y)(3x-y)$

$= 9x^2 - 3xy - 3xy + y^2$, FOIL

$= 9x^2 - 6xy + y^2$, Collected like terms

b) $(3x+y)(3x-y)$

Soln $= 9x^2 \underbrace{- 3xy + 3xy}_{=0} - y^2$, FOIL

$= 9x^2 - y^2$, Collected likes

P174 # (1-9) every other letter

(11-13) ac

(15, 16, 19, 20, 22-24) every other letter

Enrichment : Optional

W.S. " Simplifying Rational Expressions"

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Unit 3 – Toolkit – Polynomials: Factoring & Expanding

Toolkit aka Smartsheet design.

You should know and apply the following:

- Binomial products using the distributive property or FOIL.
  i.e. $(a+b)(c+d) = ac + ad + bc + bd$

- "**S**quare, **D**ouble the product of the two terms, **S**quare" pattern as a unique efficient binomial product.
  i.e. $(a \pm b)^2 = a^2 \pm 2ab + b^2$

- Unique binomial product of sum and difference binomials aka "**F**irst and **L**ast" pattern.
  $(a+b)(a-b) = a^2 - b^2$

- Common factor first
  Ex. $2x^2 - 18 = 2(x^2 - 9)$

- Group factoring.

- Sum and product trinomial factoring.
  i.e. $ax^2 + bx + c, \textit{if } a = 1$

- Tricky trinomial factoring using either the decomposition process or the guess and check process.
  i.e. $ax^2 + bx + c, \textit{if } a \neq 1$

- Factoring a difference of two squares.
  i.e. $(a)^2 - (b)^2 = (a+b)(a-b)$

MPM2DI

Simplifying Rational Expressions
Application of Factoring Polynomials

1. Simplify.

a) $\frac{2x^2-18}{4x+12}$ b) $\frac{3x^2-24x+36}{2x^2-x-6}$ c) $\frac{5x^2-25x}{3x^3-75x}$ d) $\frac{x^2+5x-24}{-3+x}$

e) $\frac{-x^2+8x-16}{x^3-4x^2}$ f) $\frac{49x-x^3}{7-6x-x^2}$ g) $\frac{a^2+11ab+18b^2}{a^2b+9ab^2}$ h) $\frac{15a^5b(5+a)}{6a^2b^3(a+5)}$

i) $\frac{4a^3b^4(a^2+4a-12)}{28a^4b^4(6+a)}$ j) $\frac{a^4-8a^3b}{a^3-64ab^2}$ k) $\frac{4a^2+8ab-12b^2}{6a^2-12ab+6b^2}$ l) $\frac{10a^3b+10a^2b}{4a^2b^3+2ab^3}$

Answers:

a) $\frac{(x-3)}{2}$ b) $\frac{3(x-6)}{(2x+3)}$ c) $\frac{5}{3(x+5)}$ d) $(x+8)$ e) $\frac{-(x-4)}{x^2}$ f) $\frac{x(x-7)}{(x-1)}$

g) $\frac{(a+2b)}{ab}$ h) $\frac{5a^3}{2b^2}$ i) $\frac{(a-2)}{7a}$ j) $\frac{a^2}{(a+8b)}$ k) $\frac{2(a+3b)}{3(a-b)}$ l) $\frac{5a(a+1)}{b^2(2a+1)}$