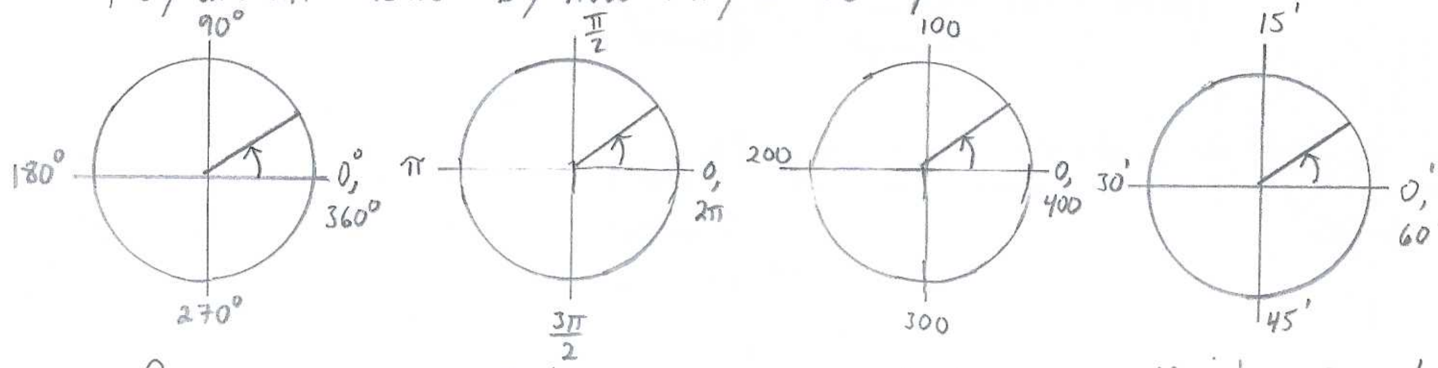
A 6.9 * Radian Measure *

Remark$_1$: There are at least 3 common sets of math units that specify angle measure; they are all related by how they divide up a circle.

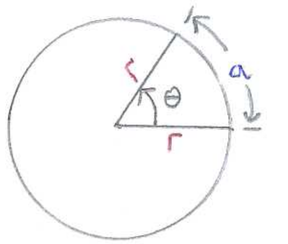


Degrees | Radians aka rads | Gradients aka grades aka gons | Minutes + Seconds

Def'n$_1$: Angle $\theta = 1$ radian, if the central angle θ of the circle traces out an arc, a, whose length is equal to the radius of the circle, r.

"This lesson sits on this definition"

"Angle θ is a measure of a rotational gap"



When $a = r$, then $\theta = 1$ radian.

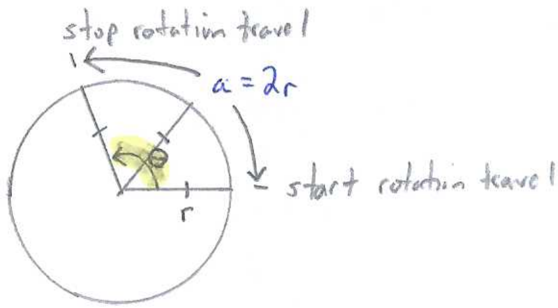
Remark$_2$: Number of radians $= \dfrac{\text{arc length}}{\text{radius length}}$. So a radian is a pure number with no units as $\frac{cm}{cm} = 1$, for example.

That is $\boxed{\theta = \dfrac{a}{r}}$.

So, $a = (\theta)(r)$, where θ is measured in radians.

arc length measure — radius measure

Note$_1$: If $\theta = 2$, then $a = 2r$ and picture is



$\theta = 2$.

Ex$_1$ How many radians are in one revolution aka one full turn?

Soln

$\because \theta = \dfrac{a}{r}$

$\therefore 360° = \dfrac{2\pi r}{r}$

$\therefore 360° = 2\pi$

So 2π radians are equal to $360°$

2π rads $= 360°$

$\boxed{\pi \text{ rads} = 180°}$

$\Rightarrow$

Physics Link: If a wheel rotates 2π radians in 10 seconds, then its angular velocity is $\frac{2\pi \text{ radians}}{10 \text{ seconds}} = \frac{\pi \text{ rads}}{5 \text{ seconds}}$

$= 0.63 \text{ rads}/1\text{sec}$

Equivalently, $\frac{360^\circ}{10 \text{ seconds}} = 36^\circ/\text{second}$

Ex₁ Convert to radians:

a) 30°

Sol'n If $\pi \text{ rads} = 180^\circ$

then $\boxed{1 \text{ rad} = \frac{180^\circ}{\pi}}$ and $\boxed{1^\circ = \frac{\pi \text{ rads}}{180}}$

$\doteq 57^\circ$

So, $1^\circ = \frac{\pi}{180}$

$\therefore 30^\circ = \frac{30\pi}{180}$

$= \frac{\pi}{6}$ ✓

b) 45°

Sol'n $1^\circ = \frac{\pi}{180}$

$\therefore 45^\circ = \frac{45\pi}{180}$

$= \frac{\pi}{4}$ ✓

c) 135°

Sol'n $1^\circ = \frac{\pi}{180}$

$\therefore 135^\circ = \frac{135\pi}{180}$

$= \frac{3\pi}{4}$ ✓

d) 82°

Sol'n $1^\circ = \frac{\pi}{180}$

$\therefore 82^\circ = \frac{82\pi}{180}$

$= \frac{41\pi}{90}$ ✓

Notation: 2π radians $= 2\pi$ rads $= 2\pi^r = 2\pi$, all equivalent to one full spin aka rotation.

Ex₂ Convert to degrees:

a) 4^r

Sol'n $\because 1 \text{ rad} = \frac{180^\circ}{\pi}$

$\therefore 4 \text{ rads} = \frac{720^\circ}{\pi}$, multiplied by 4

$\doteq 229.2^\circ$, calculator ✓

b) $\frac{5\pi}{6}$

Sol'n $\because 1 \text{ rad} = \frac{180^\circ}{\pi}$

$\therefore \frac{5\cancel{\pi}}{\cancel{6}_1} \times \frac{\cancel{180^\circ}^{30}}{\cancel{\pi}}$

$= 150^\circ$ ✓

c) 2.63 rad

Sol'n $1 \text{ rad} = \frac{180^\circ}{\pi}$

$\therefore 2.63 \text{ rad} = 2.63\left(\frac{180^\circ}{\pi}\right)$

$\doteq 150.69^\circ$ ✓

Note₂: Since radians are measures of angles, we can use trig with radians.

Ex₃ Evaluate.

a) $\tan\frac{\pi}{3}$ — In $\boxed{\text{RAD}}$ mode

$= \sqrt{3}$, calculator

b) $\sin\frac{\pi}{4}$ — In $\boxed{\text{RAD}}$ mode

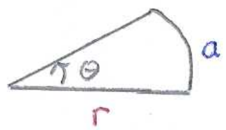
$= \frac{\sqrt{2}}{2}$, calculator

c) $\cos\frac{3\pi}{4}$ — In $\boxed{\text{RAD}}$ mode, Calculator

$= -\frac{\sqrt{2}}{2} \doteq -0.7071$

e)

Remark$_3$: $\dfrac{\text{Sector Angle}}{\text{Full Rotation Angle}} = \dfrac{\text{Arc length of a Sector}}{\text{Circumference}}$



$$\therefore \quad \boxed{\frac{\theta}{360^\circ} = \frac{a}{2\pi r}}$$

Ex$_4$ Find the arc length of a sector of a circle whose radius is 10m and the sector angle is 90°.

Sol'n

$\dfrac{\theta}{360^\circ} = \dfrac{a}{2\pi r}$, formula

$\dfrac{90^\circ}{360^\circ} = \dfrac{a}{2\pi(10)}$, sub in

$a = 5\pi$ ✓, cross multiply and then isolate a.

$\doteq 15.71\,m$

Note: This makes sense, since $\frac{1}{4} \times$ Circumference

$= \frac{1}{4} \times (2\pi r)$

$= \frac{1}{4} \times 2\pi(10)$

$= 5\pi$ ✓

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A # 9, 10.