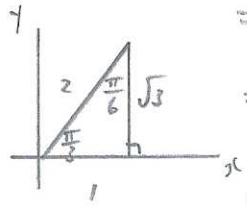


# 8.10 \* Multiple Transformations of Trigonometric Functions \*

Warmup \* 1. Evaluate. Exact answers.

a)  $\cos^2 \frac{\pi}{3} - \sin^2 \frac{\pi}{3}$

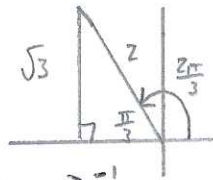
Sol'n  $= (\cos \frac{\pi}{3})^2 - (\sin \frac{\pi}{3})^2$   
 $= (\frac{1}{2})^2 - (\frac{\sqrt{3}}{2})^2$   
 $= \frac{1}{4} - \frac{3}{4}$   
 $= -\frac{2}{4}$   
 $= -\frac{1}{2} \checkmark$



RAA =  $\frac{\pi}{3}$

b)  $\sec \frac{2\pi}{3}$

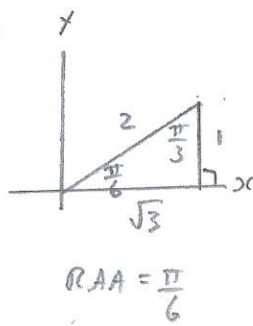
Sol'n  $= \frac{1}{\cos \frac{2\pi}{3}}$   
 $= \frac{1}{(-\frac{1}{2})}$   
 $= -2 \checkmark$



\* RAA =  $\frac{\pi}{3}$

c)  $2(\cot \frac{\pi}{6})^2 - 1$

Sol'n  $2\left(\frac{1}{\tan \frac{\pi}{6}}\right)^2 - 1$   
 $= 2\left(\frac{1}{\frac{1}{2}}\right)^2 - 1$   
 $= 2\left(\frac{1}{4}\right)^{-1} - 1$   
 $= \frac{2}{4} - 1 \times 4$   
 $= \frac{2}{4} - \frac{4}{4}$   
 $= -\frac{1}{2} \checkmark$



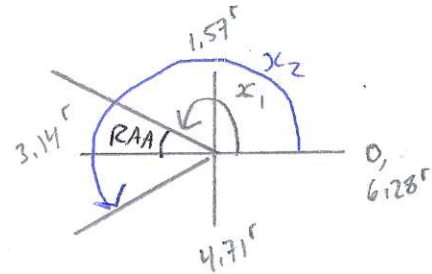
RAA =  $\frac{\pi}{6}$

2. Solve  $\cos x = -0.82$ ,  $0 \leq x \leq 2\pi$

Sol'n  $x = \cos^{-1} -0.82$   
 $x_1 = 2.5322$ , in radian mode on calculator

$\therefore$  RAA =  $\pi - 2.5322$   
 $\hat{=} 0.6094$

$\therefore x_2 = \pi + \text{RAA}$ , cosine ratio is negative in bottom left window, the T-Quad  
 $= \pi + 0.6094$   
 $\hat{=} 3.751$



So for  $x$  angle in between 0 and  $2\pi$  radians,  $\cos x = -0.82$  when  $x = 2.53^\circ$  or  $3.75^\circ$

Note: To plot both Phase Shift and Critical Interval spaced key points on corners of grid squares, for  $k$ 's not involving  $\pi$ , consider using Scale =  $\frac{\pi}{\text{L.C.M of C.I. and P.S. denoms}}$

For ex: a) If C.I. =  $\frac{\pi}{3}$  and P.S. is  $\frac{\pi}{2}$ ,  $\therefore$  Scale =  $\frac{\pi}{\text{a multiple of 3 and 2}}$ ; like  $\frac{\pi}{6}, \frac{\pi}{12}, \frac{\pi}{18}, \dots$

b) If C.I. =  $\frac{\pi}{2}$  and P.S. is  $\frac{\pi}{4}$ ,  $\therefore$  Scale =  $\frac{\pi}{\text{a multiple of 2 and 4}}$ ; like  $\frac{\pi}{4}, \frac{\pi}{8}, \dots$

# Multiple Transformations of Trigonometric Functions

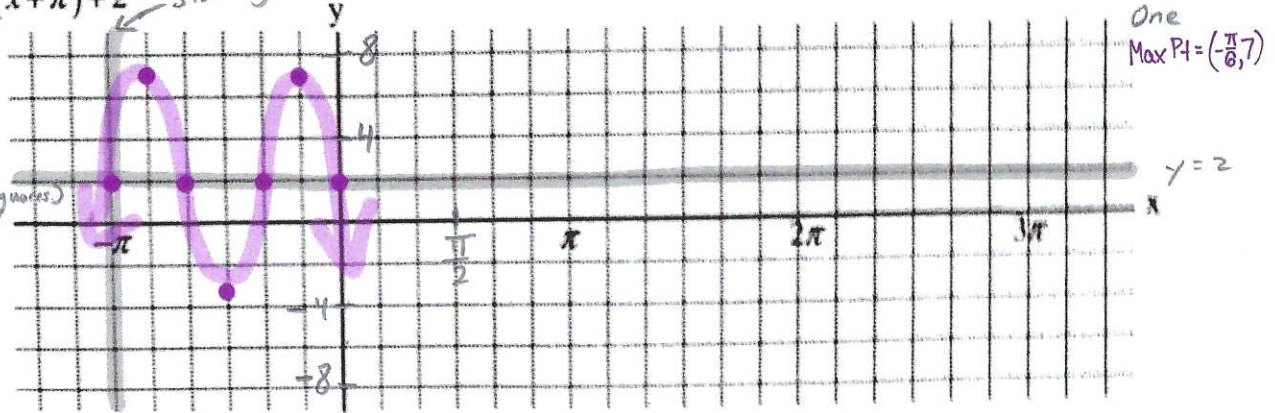
Tip:

- 1<sup>st</sup> · Slide axis (Translations)
- 2<sup>nd</sup> · Critical Points
- 3<sup>rd</sup> · Join points

**Example 1:** Sketch the graph of each of the following:

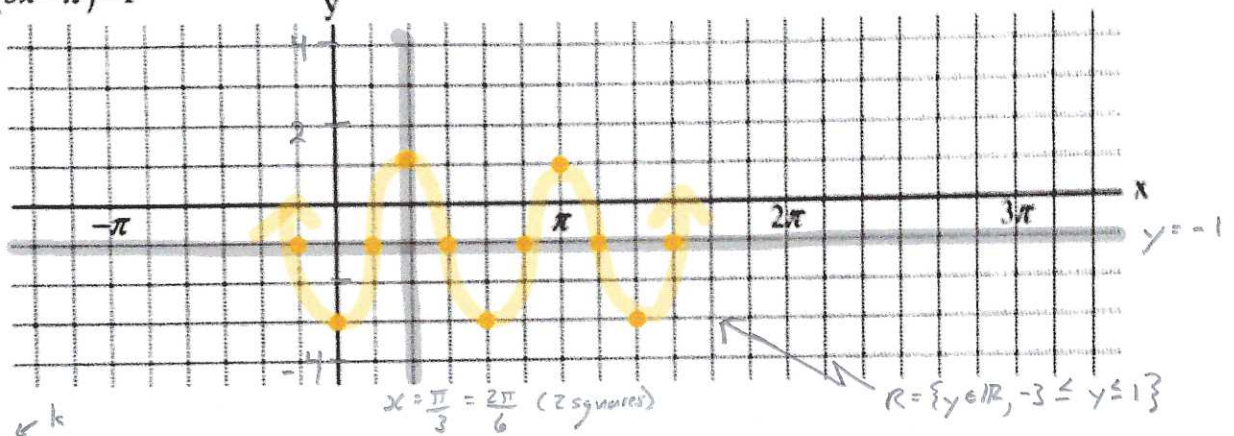
a)  $y = 5\sin 3(x + \pi) + 2$     "starting line" → Axis, Max, Axis, Min, Axis

- Amp = 5
- Period =  $\frac{2\pi}{3}$
- P.S. =  $-\pi$ , (Left 6 squares)
- Sinusoidal axis:  $y = 2$
- C.I. =  $\frac{2\pi}{3} = \frac{\pi}{6}$
- ∴ C.I. =  $\frac{\text{Per}}{4}$



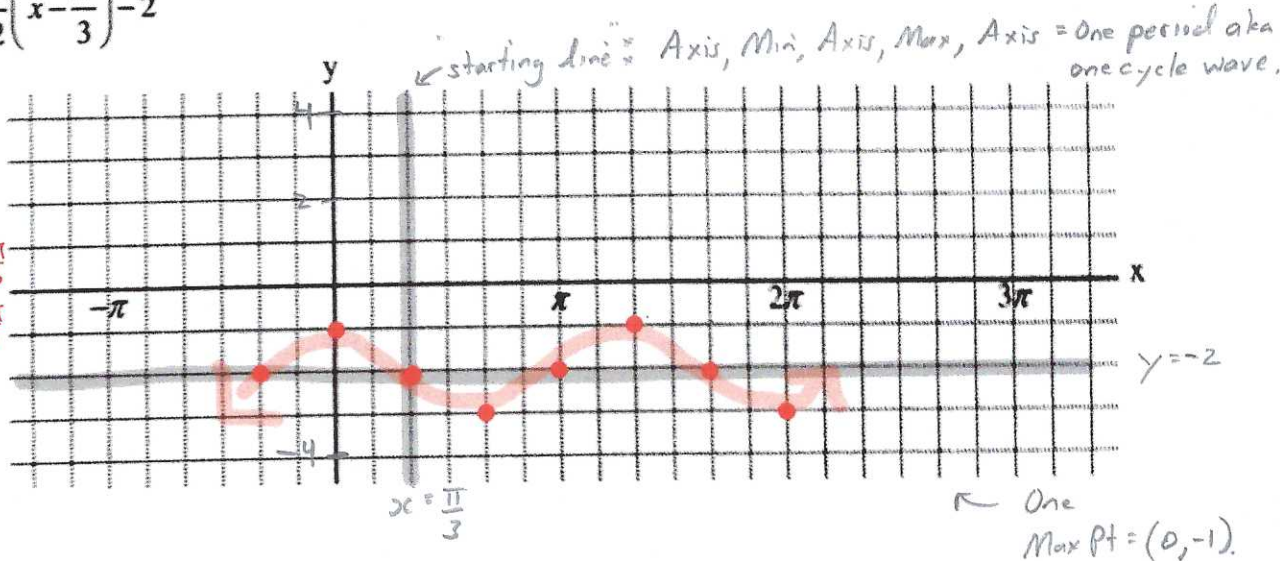
b)  $y = 2\cos(3x - \pi) - 1$     Recall: Factor out 3 to correctly read the phase shift quantity.

- $y = 2\cos 3(x - \frac{\pi}{3}) - 1$
- Amp = 2
- P.S. =  $\frac{\pi}{3}$
- Period =  $\frac{2\pi}{3}$
- S.A.:  $y = -1$
- C.I. =  $\frac{2\pi}{3} = \frac{\pi}{6}$



c)  $y = -\sin \frac{3}{2}(x - \frac{\pi}{3}) - 2$

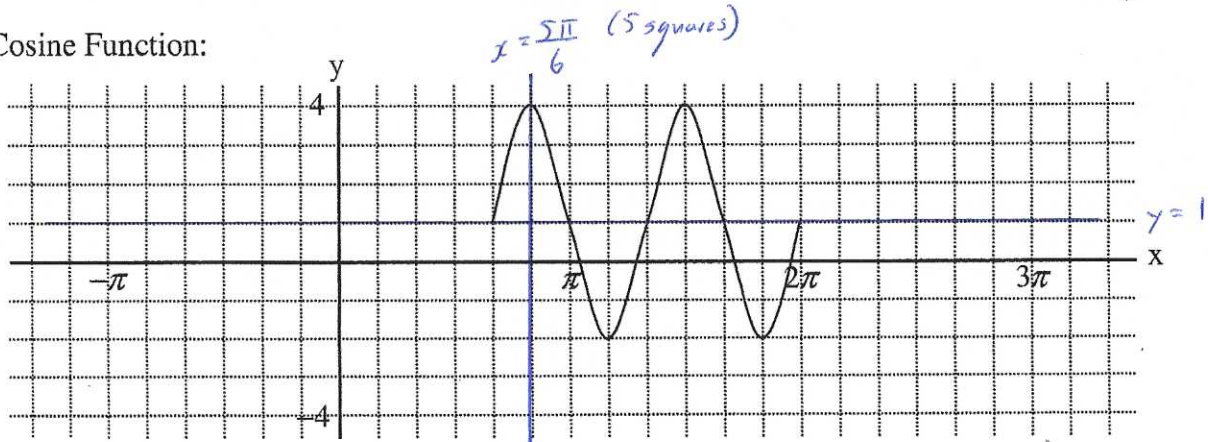
- S.A.:  $y = -2$
- Reflection in x-axis
- P.S. =  $\frac{\pi}{3}$
- Period =  $\frac{2\pi}{3/2} = \frac{4\pi}{3} = \frac{8\pi}{6}$
- C.I. =  $\frac{4\pi}{3} = \frac{4\pi}{12} = \frac{2\pi}{6}$
- ∴  $\frac{1}{4} \times \frac{2\pi}{6} = \frac{1}{3} \times \frac{1}{4}$



Example 2: Write the equations for the function graphed below.

$$y = a \cos k(x-p) + q$$

(a) Cosine Function:



$$\begin{aligned} \text{Amp} &= (\text{Max} - \text{Min}) \div 2 \\ &= ((4) - (-2)) \div 2 \\ &= (6) \div 2 \\ &= 3 \end{aligned}$$

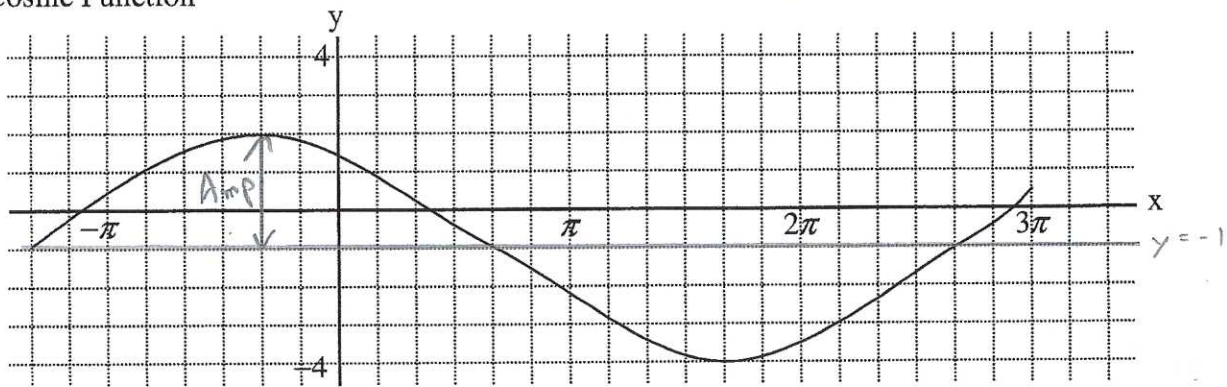
$$\begin{aligned} \text{P.S.} &= \frac{5\pi}{6} \\ \text{To Find } k & \end{aligned}$$

$$\begin{aligned} \text{Period} &= \frac{4\pi}{6} \text{ (4 squares)} \\ \text{Period} &= \frac{4\pi}{6} = \frac{2\pi}{k} \\ \text{From given graph} & \quad \therefore k=3 \end{aligned}$$

Formula for all sinusoids

$$\begin{aligned} \checkmark \text{ So, } y &= 3 \cos 3(x - \frac{5\pi}{6}) + 1 \text{ or} \\ \checkmark y &= -3 \cos 3(x - \frac{7\pi}{6}) + 1 \\ \text{or ...} & \end{aligned}$$

(b) Cosine Function

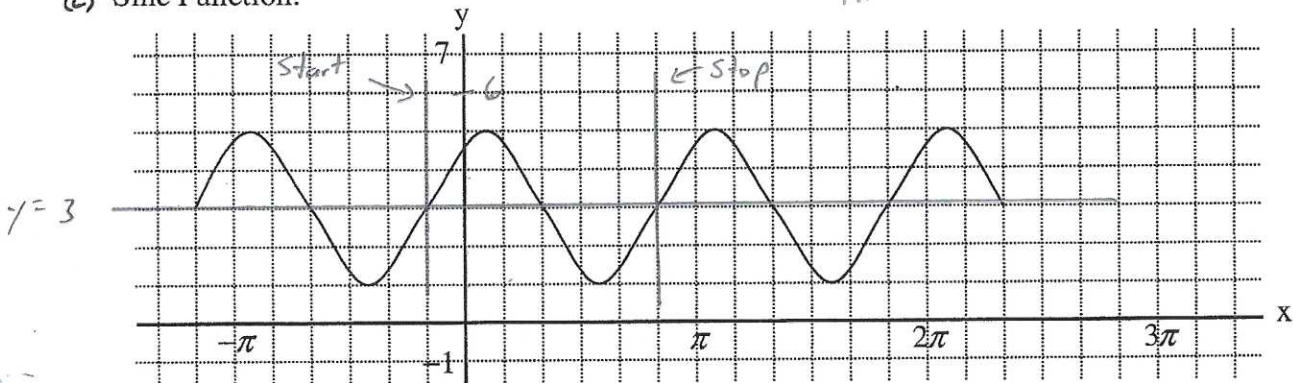


$$\begin{aligned} \text{S.A.} & \checkmark y = -1 \\ \text{Amp} & \checkmark 3 \\ \text{P.S.} & \checkmark -\frac{2\pi}{6} = -\frac{\pi}{3} \\ & \text{"left"} \end{aligned}$$

$$\begin{aligned} \text{Per} &= 4\pi \\ \text{Find } k & \quad \frac{2\pi}{k} = 4\pi \\ 4\pi k &= 2\pi, \text{ X-multiply} \\ k &= \frac{2\pi}{4\pi} = \frac{1}{2}, \text{ divide} \end{aligned}$$

$$\checkmark \text{ So, } y = 3 \cos \frac{1}{2}(x + \frac{\pi}{3}) - 1$$

(c) Sine Function:



$$\begin{aligned} \text{S.A.} & \checkmark y = 3 \\ \text{Amp} & \checkmark (\text{Max} - \text{Min}) \div 2 = 2 \\ \text{P.S.} & \checkmark -\frac{\pi}{6} \\ & \text{"left"} \end{aligned}$$

$$\begin{aligned} \text{Period} &= \frac{6\pi}{6} = \pi \\ \text{Find } k & \quad \frac{2\pi}{k} = \pi \\ \therefore k &= 2 \end{aligned}$$

$$\checkmark \text{ So, } y = 2 \sin 2(x + \frac{\pi}{6}) + 3 \text{ or ...}$$

A p 355 #1, 2, Sac, 6bd, 8a, 9c  
+ Grimo \* Translating and Scaling Sine and Cosine Functions