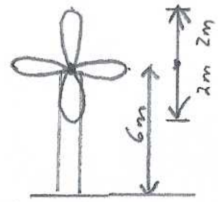
A 8.12 * Applications *



$y = a\cos k(x-p)+q$

OR $y = a\sin k(x-p)+q$

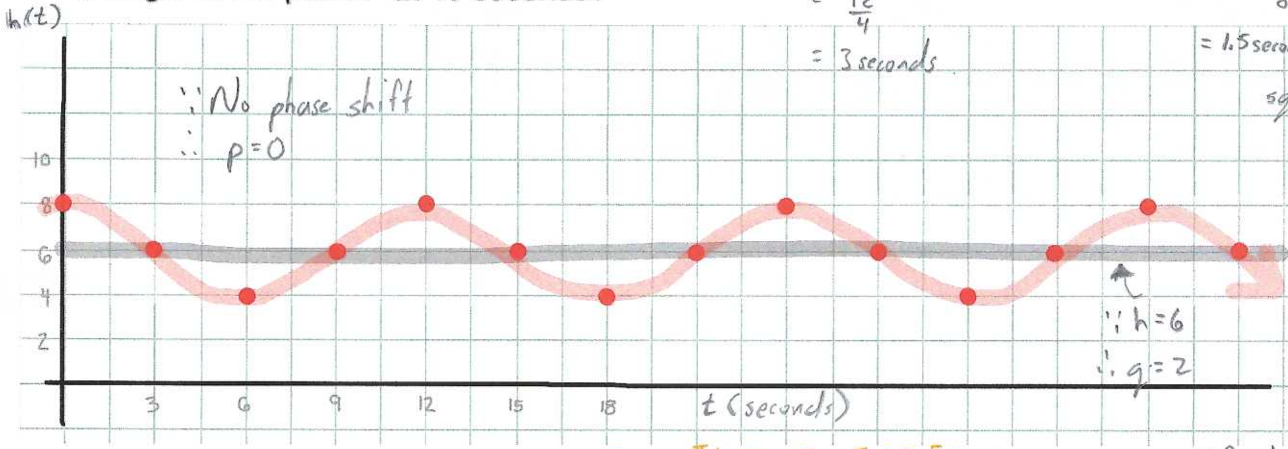
Example₁: A small windmill has its center 6 metres above the ground and blades 2 metres in length. In a steady wind, a point P at the tip of one blade makes a complete rotation in 12 seconds.

a) If the rotation begins at the highest possible point, determine a function that gives the height of point P above the ground at time *t*.

b) What is the length of the point P at 40 seconds?

Scale $= \frac{Per}{4}$ $= \frac{12}{4}$ = 3 seconds

Note: We built more room for the C.I.: $\frac{12 \text{ secs}}{8}$ = 1.5 seconds / square



a) Amp: 2

Period = 12

C.I. = 3

P.S.: X

$\because Per = \frac{2\pi}{k}$ $\therefore k = \frac{2\pi}{12}$

✓ $k = \frac{\pi}{6}$

$y = 2\cos\frac{\pi}{6}(t+0)+6$

b) $h = 2\cos\frac{\pi}{6}(40)+6$

$= 5$ m

∴ When t = 40s, the blade is 5m above ground zero

c) Determine when pt P is 7m above the ground

$7 = 2\cos\frac{\pi}{6}t+6$

$1 = 2\cos\frac{\pi}{6}t$

$\frac{1}{2} = \cos\frac{\pi}{6}t$

$\frac{\pi}{6}t = \cos^{-1}\left(\frac{1}{2}\right)$

$\frac{\pi}{6}t = \frac{\pi}{3}$, QI ; $\frac{5}{3}\pi$, QIV

$t = \frac{\frac{\pi}{3}}{\frac{\pi}{6}} = 2s$ $t = \frac{\frac{5\pi}{3}}{\frac{\pi}{6}} = 10s$

∴ t = 2 + 12 ← Period = 12

= 14 secs

∴ t = 10 + 12

= 22 secs

Example₂: In the bay of Fundy, docks have been built high above the ocean floor because of the extremely high tides. On an average day, the depth of the water around a harbor dock changes from 1.5 metres at low tide at 03:00 hours to 15.5 metres at high tide at 09:00. The data recorded in the table below show the depth of water in a 24 hour period.

Time (hrs)	0	3	6	9	12	15	18	21	24
Depth (metres)	8.5	1.5	8.5	15.5	8.5	1.5	8.5	15.5	8.5

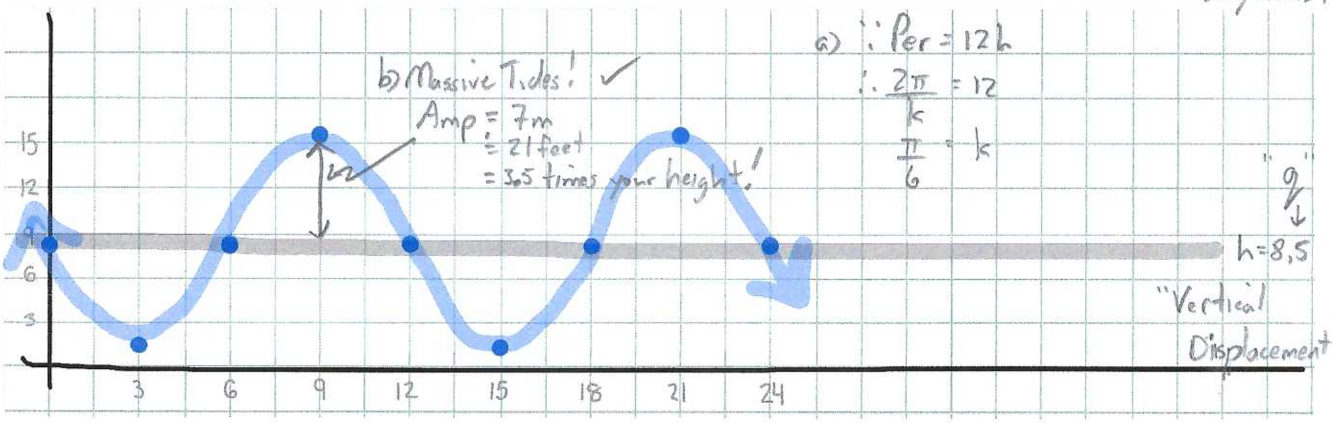
a) Write an equation for the depth of the water at time *t*.

b) What is the significance of the amplitude of this function?

c) Will it be safe for a ship to enter the harbor at 16:00, if the ship requires 3.5 m of water?

Scale $= \frac{Per}{4}$ = 3 hours.

Note: We built more room for the C.I.: 3 hours every 2 squares.



a) ∵ Per = 12h

$\therefore \frac{2\pi}{k} = 12$

$\frac{\pi}{6} = k$

max, min

Amp = (15.5 − 1.5) ÷ 2

= 7

a) $d = -7\sin\frac{\pi}{6}(t+0)+8.5$

sinusoidal opens down; no phase shift; q

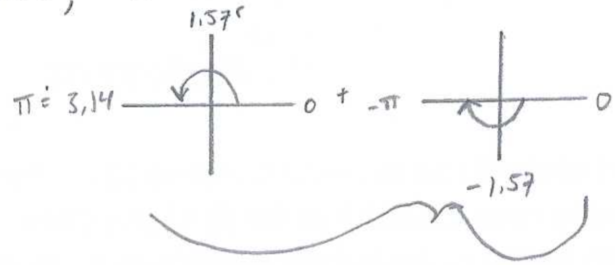
c) $d = -7\sin\frac{\pi}{6}(16)+8.5$

d = 2.44m

∵ 2.44m less than 3.5m

∴ Unsafe. ✓

Example$_3$: Solve $\cos\theta = -0.623$, $0 \leq \theta \leq \pi$ and $-\pi \leq \theta \leq 0$.



aka $-\pi \leq \theta \leq \pi$

Sol'n

$\theta = \cos^{-1}(-0.623)$

$\theta_1 = 2.2434^r$

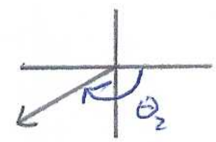
$\therefore RAA = \pi - 2.2434$

$\doteq 0.8982$

$\therefore \theta_2 = -(\pi - RAA)$

$= -(\pi - 0.8982)$

$= -2.2434$



So $\theta \doteq 2.24^r, -2.24^r$

✓ ✓

A p369 #1 ⇒ Note: $h = 1.8 \sin \dfrac{2\pi(t-4.00)}{12.4} + 3.1$ is equivalent to

$h = 1.8 \sin \dfrac{2\pi}{12.4}(t-4.00) + 3.1$.

aka

$h = 1.8 \sin \dfrac{\pi}{6.2}(t-4.00) + 3.1$

An old fashioned way of clearly and quickly showing the Period = 12.4 h

#3

#4ab

#6, 7, 8.