# 4.4 * $x = -\frac{b}{2a}$ * aka * Complete the Square Part II *

Warmup: Write in vertex form $y = a(x-h)^2 + k$.

a) $y = x^2 + 4x - 3$

Sol'n
$y = x^2 + 4x + 4 - 4 - 3$

$+\left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2$

$y = (x+2)^2 - 4 - 3$

$y = (x+2)^2 - 7$

b) $y = -2x^2 + 8x - 5$

Sol'n
$y = -2(x^2 - 4x) - 5$, step #1 "a" to front

$y = -2(x^2 - 4x + 4 - 4) - 5$, step #2 special number

$y = -2(x^2 - 4x + 4) - 4(-2) - 5$ step #3 pull

$y = -2(x-2)^2 \underbrace{- 4(-2) - 5}$ step #4 factor

$y = -2(x-2)^2 + 3$ step #4 tidy up.

Remark: Formula: Max/Min occurs when $\boxed{x = -\frac{b}{2a}}$

Max/Min value is calculated by substituting $x = -\frac{b}{2a}$ into the given equation of the parabola to find y.

Ex, State the max/min value of y and the value of x when it occurs.

a) $x^2 + 8x + 27$

Sol'n Minimum value when $x = -\frac{b}{2a}$

$= \frac{-(8)}{2(1)}$

$= -4$

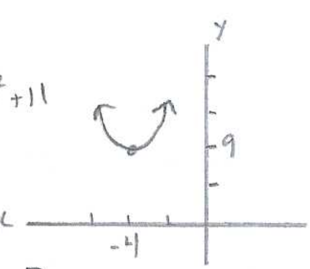
"Because a = 1 which is greater than zero. Parabola graph opens Up."

Min value is $y = x^2 + 8x + 27$

$y = (-4)^2 + 8(-4) + 27$

$y = 11$

$y = (x+4)^2 + 11$




Compare to yesterday, $Ex_2$ a)

Works!

b) $y = 4x^2 + 24x + 38$

Sol'n Min value when $x = -\frac{b}{2a}$

$= \frac{-(24)}{2(4)}$

$= -3$

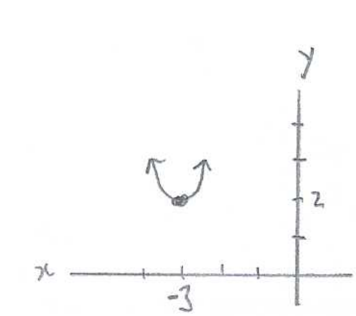
Min value is $y = 4(-3)^2 + 24(-3) + 38$

$y = 2$

"Comes along for the ride"

$y = 4(x+3)^2 + 2$

"Think opposite"




Compare to yesterday, $Ex_3$ a)

Formula Works!

e)

**Remark$_2$**: Just watch... the development of our fast and accurate $x = \frac{-b}{2a}$ formula.

C.T.S. for $y = ax^2 + bx + c$

**Sol'n**

$y = a(x^2 + \frac{b}{a}x) + c$, step #1 "a" to front

$y = a(x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 - (\frac{b}{2a})^2) + c$, step #2 special number to build perfect square trinomial

$y = a(x^2 + \frac{b}{a}x + (\frac{b}{2a})^2) - (\frac{b}{2a})^2(a) + c$, step #3 pull.

$y = a(x + \frac{b}{2a})^2 - (\frac{b^2}{4a^2})(a) + c$, step #4 factor.

$y = a(x + \frac{b}{2a})^2 - \frac{b^2}{4a} + c$,

$y = a(x + \frac{b}{2a})^2 - \frac{b^2}{4a} + \frac{4ac}{4a}$, step #4 tidy up, and common denom build.

$y = a(x + \frac{b}{2a})^2 + \frac{4ac}{4a} - \frac{b^2}{4a}$, shuffle up

$y = a(x + \frac{b}{2a})^2 + \frac{4ac - b^2}{4a}$, done

$$\text{Vertex}\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$$

"easy to recall" "tough to recall, so don't ☺, Just substitute in."

**Ex$_2$** Determine the axis of symmetry and vertex of $y = -2x^2 + 4x + 1$.

**Sol'n$_1$** "$x = \frac{-b}{2a}$ process"

$a = -2$

$b = 4$

$c = 1$

Max value when $x = \frac{-b}{2a}$

$x = \frac{-(4)}{2(-2)}$

$= 1$

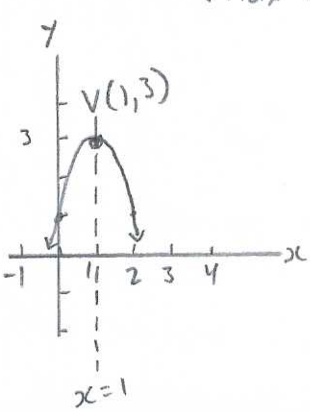
Max value is $y = -2(1)^2 + 4(1) + 1$

$= -2 + 4 + 1$

$= 3$

So vertex is $V(1,3)$ ✓

and A of S is $x = 1$.




$y = -2(x-1)^2 + 3$

**Sol'n$_2$** "C.T.S. process"

$y = -2(x^2 - 2x) + 1$, step #1

$y = -2(x^2 - 2x + 1 - 1) + 1$, step #2

$+(\frac{-2}{2})^2 - (\frac{-2}{2})^2$

$y = -2(x^2 - 2x + 1) - 1(-2) + 1$, step #3

$y = -2(x-1)^2 - 1(-2) + 1$, step #4

$y = -2(x-1)^2 + 3$, step #4.

$V(1,3)$ ✓

and A of Sym is $x = 1$

p 235 #(8,9)ac

#12ace