

# \* Unit 5 Review: Trig Functions \*

Warmup: 1. Solve for  $0 \leq \theta < 2\pi$ . aka Angle is in between  $0^\circ$  and  $2\pi$  radians.

a)  $\sin^2 \theta - \sin \theta = 0$

Sol'n  $\sin \theta (\sin \theta - 1) = 0$

$\therefore \sin \theta = 0$  or  $\sin \theta = 1$

So  $\theta = 0, \pi, 2\pi$

$\sin \theta = 1$

So  $\theta = \frac{\pi}{2}$



So S.S. =  $\{0, \frac{\pi}{2}, \pi, 2\pi\}$

b)  $2\cos^2 \theta - 1 = 0$

Sol'n  $\cos^2 \theta = \frac{1}{2}$

$\cos \theta = \frac{1}{\sqrt{2}}$  or  $\cos \theta = -\frac{1}{\sqrt{2}}$

$\therefore \theta_1 = \frac{\pi}{4}$

$\therefore \theta_1 = \pi - \frac{\pi}{4}$

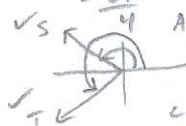
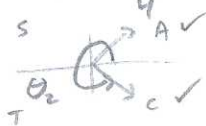
$= \frac{3\pi}{4}$

$\theta_2 = 2\pi - \frac{\pi}{4}$

$\theta_2 = \pi + \frac{\pi}{4}$

$= \frac{7\pi}{4}$

$= \frac{5\pi}{4}$



So S.S. =  $\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

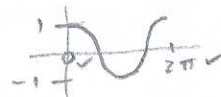
c)  $\sec x - 3 = -2$

$+3 \quad +3$

Sol'n  $\sec x = 1$

So  $\cos x = 1$ , flip-flip.

$\therefore x = 0, 2\pi$



d)  $\csc^2 \theta - \csc \theta - 6 = 0$

Sol'n  $k^2 - k - 6 = 0$ , helpful substitution:  $k = \csc \theta$ .

$(k-3)(k+2) = 0$

$\therefore k-3=0$  or  $k+2=0$

$\therefore k=3$  or  $k=-2$

$\therefore \csc \theta = 3$  or  $\csc \theta = -2$

$\therefore \sin \theta = \frac{1}{3}$  or  $\sin \theta = -\frac{1}{2}$ , flip-flip

$\therefore \theta = \sin^{-1}(\frac{1}{3})$

$\theta_1 = 0.3398^\circ$  RAA

OR

$\theta_2 = \pi - \text{RAA}$

$= 2.8018^\circ$



OR  $\theta = \sin^{-1}(-\frac{1}{2})$

$\theta = -\frac{\pi}{6}$

So RAA =  $\frac{\pi}{6}$

So  $\theta_1 = \pi + \text{RAA}$

$= \frac{7\pi}{6}$

OR  $\theta_2 = 2\pi - \text{RAA}$

$= \frac{11\pi}{6}$

$= \frac{11\pi}{6}$

So S.S. =  $\{0.34^\circ, 2.80^\circ, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

Remark: You should know and apply:

MHF4UI

Unit 5 Review: Trigonometry Functions

May 2020

## Units 5 - Trig Functions Essential Learnings

I can:

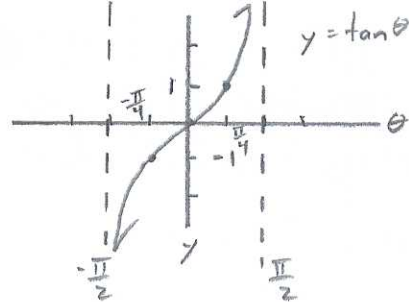
- convert between degrees and radian measure.
- evaluate, with & without technology, trigonometric functions and their reciprocal functions using radians
- use radian measure working with special triangles, angles in standard position and the CAST Rule.
- identify the key features of trigonometric function graphs: domain, range, period.
- carefully graph the primary trigonometric functions using radian angle measure.
- apply transformations to and graph transformations of the basic trigonometric functions graphs.
- set x-axis and y-axis scales to model real world phenomena (windmill, tides, bicycles).
- solve linear trig and quadratic trig equations

Recall<sub>1</sub>:

$$y = a \sin k(x-p) + q$$

Vertical stretch  
 Vertical reflection  
 Horizontal Reflection  
 Horizontal stretch  
 Vertical Displacement

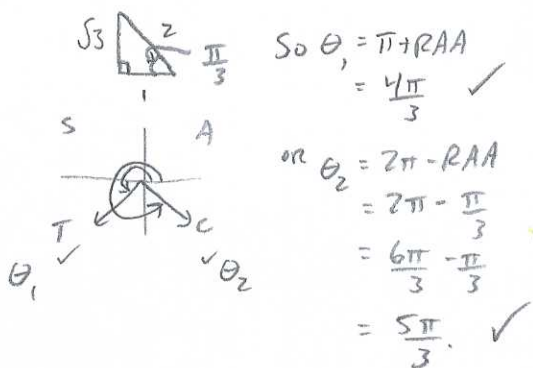
and.



Ex<sub>1</sub> Provide exact answers.

a)  $\sin x = -\frac{\sqrt{3}}{2}, 0 \leq x \leq 2\pi$

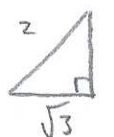
Sol<sub>1</sub> RAA =  $\frac{\pi}{3}$



b)  $\tan^2 \theta = \frac{1}{3}, 0 \leq \theta \leq 2\pi$

Sol<sub>1</sub>  $\tan \theta = \pm \sqrt{\frac{1}{3}}$

$\tan \theta = \pm \frac{1}{\sqrt{3}}$



RAA =  $\frac{\pi}{6}$

clues:  
Just List All 4 Quadrant Rotations

$\therefore \theta_1 = \frac{\pi}{6}$  ✓

$\theta_2 = \frac{5\pi}{6}$  ✓

$\theta_3 = \frac{7\pi}{6}$  ✓

$\theta_4 = \frac{11\pi}{6}$  ✓

✓ Finish 2 Gizmos

✓ P372<sup>2</sup> (1-4) every other letter, (6,7)ac, 9b, 12

✓ + "Trigonometry I-Review" W.S.

Remark<sub>2</sub>:

	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1 = \frac{\sqrt{4}}{2}$
$\cos \theta$	$1 = \frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{2} = \frac{\sqrt{1}}{2}$	$0 = \frac{\sqrt{0}}{2}$
$\tan \theta$	$= 0$	$= \frac{1}{\sqrt{3}}$	$= 1$	$= \sqrt{3}$	$= \infty$

$\cos \theta$

$\tan \theta$

$\frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2} = \sin \theta \div \cos \theta$  for  $\theta = 45^\circ = 1$   
 $\frac{\sqrt{4}}{2} \div \frac{\sqrt{0}}{2} = \sin \theta \div \cos \theta$  for  $\theta = 90^\circ = \infty$

Having Fun with number patterns...

And then recall  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ , segue to Unit 6...

## Trigonometry I - Review

1. Convert each degree to radian measure in terms of  $\pi$ .

- a)  $45^\circ$                       b)  $300^\circ$                       c)  $-210^\circ$

2. Convert each radian measure to degrees. If the answer is not exact, round correctly to one decimal place.

a) $\frac{5\pi}{6}$ rad	b) $\frac{9}{4}\pi$ rad	c) 6 rad	d) $3.5\pi$ rad
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3. Determine the related acute angle associated with each of the following standard position angles. Provide a clearly labelled sketch.

- a)  $\frac{2\pi}{3}$                       b)  $-\frac{7\pi}{6}$                       c)  $\frac{7\pi}{4}$                       d) 4.25

4.  $\theta$  is in standard position with its terminal arm in the quadrant 4,

$\sin\theta = -\frac{12}{13}$ , and  $0 < \theta < 2\pi$ . Determine the exact value of  $\cos\theta$  and  $\tan\theta$ . Provide a clearly labelled sketch.

5. For each of the following, (i) sketch the standard position angle, (ii) determine the related acute angle, and (iii) determine the exact value of the specified trigonometric ratio. Do not use a calculator.

- a)  $\tan\left(\frac{5\pi}{4}\right)$                       b)  $\sin\left(\frac{11\pi}{6}\right)$                       c)  $\cos\left(\frac{2\pi}{3}\right)$                       d)  $\tan\left(\frac{-7\pi}{2}\right)$

6. Solve each of the following equations to two decimal places,  $0 \leq \theta \leq 2\pi$ .

- a)  $\sin\theta = -0.1357$                       b)  $\cos\theta = 0.2468$                       c)  $\cot\theta = -3$

7. Solve each of the following equations. Give exact answers only,  $0 \leq \theta \leq 2\pi$ .

- a)  $\cos\theta = -\frac{\sqrt{3}}{2}$                       b)  $\csc\theta = \sqrt{2}$                       c)  $\tan\theta = -1$

8. Solve each of the following equations.

a)  $3\cos\left(x + \frac{\pi}{3}\right) + 3 = 1, 0 \leq x \leq 2\pi$ , correct to two decimal places.

b)  $\tan\left(x + \frac{5\pi}{6}\right) = \sqrt{3}, 0 \leq x \leq 2\pi$ , exact answers.

9. Solve  $2\cos^2 x + \cos x = 0$  for  $x, 0 \leq x \leq 2\pi$ . Give exact answers.

10. For each of the following functions,

- i) determine the amplitude, period, phase shift and vertical shift;  
 ii) graph at least one complete period; and  
 iii) state the domain and range.

a)  $y = 3\cos\frac{1}{4}\left(\theta + \frac{\pi}{3}\right) - 2$                       b)  $y = \frac{1}{2}\sin\left(3\theta + \frac{3\pi}{4}\right) + 2.5$

11. Graph the following functions for  $0 \leq x \leq 2\pi$ .

a)  $y = \tan\left(2\left(x + \frac{\pi}{4}\right)\right)$

12. Determine the equation of a cosine function with maximum value 5, minimum value -9, period  $\frac{2\pi}{5}$  and

phase shift  $\frac{\pi}{4}$ .

13. Determine the following features from the graph shown:

Amplitude = \_\_\_\_\_

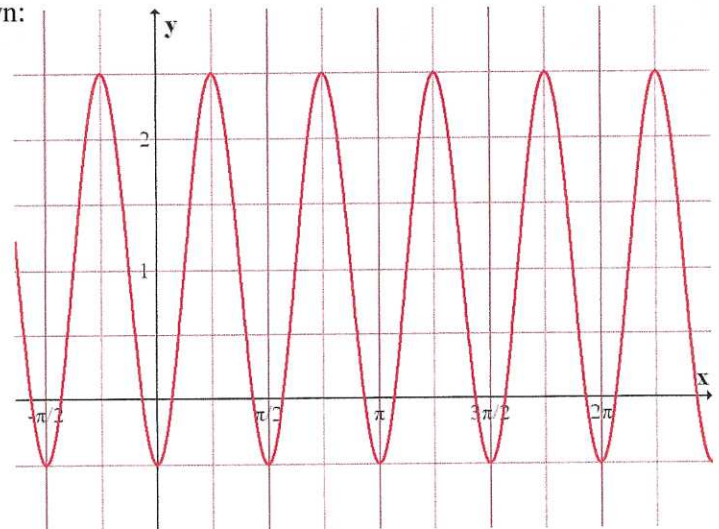
Period = \_\_\_\_\_

Phase Shift = \_\_\_\_\_

Vertical Shift. = \_\_\_\_\_

**Cosine Function Equation:**

\_\_\_\_\_



14. Given some function,  $d = 4 \sin \frac{\pi}{10}(t - 3) + 6$ :

- Determine the domain and range.
- Determine the amplitude, period, phase shift and vertical shift.
- Determine  $d$  when  $t = 11$ , answer accurate to one decimal place.
- Determine all values of  $t$  such that  $d$  is 8. State the exact answers.

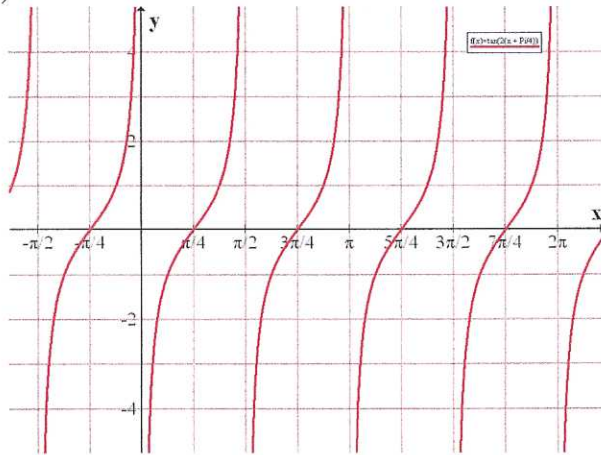
15. A Ferris wheel with a diameter of 36 m rotates three times every two minutes. Passengers get on at the lowest point which is 6 m above the ground.

- Draw a graph showing the height of the rider above the ground during the first two minutes.
- Write an equation which expresses your height as a function of time on the ride.
- Calculate the **exact** height of the rider above the ground after 25 s.
- At what times in the first rotation will the rider be 10 m or less above the ground?

Answers

1. a)  $\frac{\pi}{4}$  b)  $\frac{5\pi}{3}$  c)  $-\frac{7\pi}{6}$  2. a)  $150^\circ$  b)  $405^\circ$  c)  $343.8^\circ$  d)  $630^\circ$  3a)  $\frac{\pi}{3}$  b)  $\frac{\pi}{6}$  c)  $\frac{\pi}{4}$  d) 1.11
4.  $\cos\theta = \frac{5}{13}$ ,  $\tan\theta = -\frac{12}{5}$  5. a) 1 b)  $-\frac{1}{2}$  c)  $-\frac{1}{2}$  d) undefined 6. a) 3.28, 6.15 b) 1.32, 4.96 c) 2.82, 5.96
7. a)  $\left\{\frac{5\pi}{6}, \frac{7\pi}{6}\right\}$  b)  $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$  c)  $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$  8. a) 1.25, 2.94 b)  $\frac{\pi}{2}, \frac{3\pi}{2}$  9.  $\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}$
10. a) i) amp = 3, period =  $8\pi$ , ps =  $-\frac{\pi}{3}$ , vs = -2  
 iii) Domain =  $\{\theta \in R\}$ , Range =  $\{y \mid -5 \leq y \leq 1, y \in R\}$   
 b) i) amp = 0.5, period =  $\frac{2}{3}\pi$ , ps =  $-\frac{\pi}{4}$ , vs = 2.5  
 iii) Domain =  $\{\theta \in R\}$ , Range =  $\{y \mid 2 \leq y \leq 3, y \in R\}$

11. a)



12.  $y = 7 \cos 5\left(x - \frac{\pi}{4}\right) - 2$
13. amp = 1.5, period =  $\frac{\pi}{2}$ , ps =  $\frac{\pi}{4}$ , vs = 1,  $y = 1.5 \cos 4\left(x - \frac{\pi}{4}\right) + 1$
14. a) Domain =  $\{t \in R\}$ , Range =  $\{d \mid 2 \leq d \leq 10, d \in R\}$  b) amp = 4, period = 20, ps = 3, vs = 6  
 c) 8.4 d)  $t = \left\{\frac{14}{3} + 20k, \frac{34}{3} + 20k, k \in I\right\}$
15. b) amp = 18, period = 40, vs = 24, if ps = 0 then  $h = -18 \cos \frac{\pi}{20}t + 24$   
 c)  $(9\sqrt{2} + 24)m$  d) from 0 to 4.3 seconds and from 35.7 to 40 seconds