5.4 * The Quadratic Formula *

Remark: If it is impossible or very difficult to factor a Quadratic Equation, we use the Quadratic Formula to solve.

Recall: To write $y = 3x^2 - 12x + 8$ into vertex form $y = a(x-p)^2 + q$, we can manipulate using the C.T.S. process **

$$y = 3(x^2 - 4x) + 8$$
$$y = 3(x^2 - 4x + 4 - 4) + 8$$
$$y = 3(x^2 - 4x + 4) - 4(3) + 8$$
$$y = 3(x-2)^2 - 4$$

In general, if $y = ax^2 + bx + c$

then $0 = ax^2 + bx + c$ for x-intercepts.

$$0 = x^2 + \frac{b}{a}x + \frac{c}{a}, \text{ ÷ by } a$$

$$0 = x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}, \text{ special number } \left(\frac{\frac{b}{a}}{2}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$0 = \underbrace{x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}} - \frac{b^2}{4a^2} + \frac{4ac}{4a^2}, \text{ built common denom for last 2 terms}$$

$$0 = \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{4ac}{4a^2}, \text{ factor}$$

$$\frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \left(x + \frac{b}{2a}\right)^2, \text{ starting to isolate } x.$$

$$\frac{b^2 - 4ac}{4a^2} = \left(x + \frac{b}{2a}\right)^2, \text{ subtract fractions}$$

$$\sqrt{\frac{b^2-4ac}{4a^2}} = \sqrt{\left(x + \frac{b}{2a}\right)^2}, \text{ square root both sides}$$

$$\pm\frac{\sqrt{b^2-4ac}}{2a} = x + \frac{b}{2a}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}, \text{ switched sides}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$\star\star \quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \star\star$$

"Just read this and try to understand at least one manipulation. You are not responsible for knowing this. Just memorize the end result formula, the handy dandy Quadratic Formula"

Ex. Solve $2x^2 + 7x + 1 = 0$ (a b c)

Sol'n "Plug and Play"

$a = 2$, $b = 7$, $c = 1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(1)}}{2(2)}$$
$$x = \frac{-7 \pm \sqrt{49-8}}{4}$$
$$x = \frac{-7 \pm \sqrt{41}}{4} \quad \checkmark$$

Ex 2 Solve $2x^2 - 9x + 6 = 0$
(a, b, c)

Soln

$a = 2$
$b = -9$
$c = 6$

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, quad formula

$\therefore x = \dfrac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(6)}}{2(2)}$, sub in

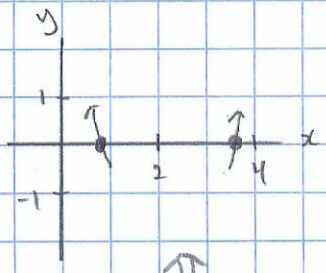
$x = \dfrac{9 \pm \sqrt{81 - 48}}{4}$

$x = \dfrac{9 \pm \sqrt{33}}{4}$

"Sub and Solve"

$\rightarrow x = \dfrac{9 + \sqrt{33}}{4} \doteq 3.69$

$\rightarrow x = \dfrac{9 - \sqrt{33}}{4} \doteq 0.81$



exact roots ⇑ approx roots ⇑ x-intercept locations ⇑

Ex 3 Solve the Quadratic Equation $x^2 - 14x - 32 = 0$
(a, b, c)

Soln $a = 1$
$b = -14$
$c = -32$

"Formula, Sub, Tidy up"

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(-32)}}{2(1)}$

$x = \dfrac{14 \pm \sqrt{196 + 128}}{2}$

$x = \dfrac{14 \pm \sqrt{324}}{2}$

$x = \dfrac{14 \pm 18}{2}$, since 324 is a perfect square ☺

$x = \dfrac{14}{2} \pm \dfrac{18}{2}$

$\rightarrow 7 + 9 = 16$

$\rightarrow 7 - 9 = -2$

⇑ exact roots.

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