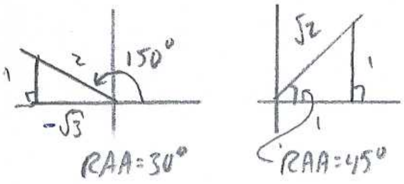
N TR. 2 *Addition Identities: Tangent*

Warmup: 1. Calculate $\sin \frac{13\pi}{12}$ exactly using a compound angle identity.



Sol'n
$= \sin 195°$
$= \sin(150° + 45°)$
$= \sin 150 \cos 45 + \sin 45 \cos 150$, sin(sum) identity stretch
$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{\sqrt{3}}{2}\right)$
$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$
$= \frac{1-\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}-\sqrt{6}}{4}$, denom rationalized. ✓

[could use calculator's decimals to check...Compare function values aka output values aka trig ratio values.]

Remark$_1$: Sum Formula For Tangent: $\boxed{\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}}$

Technically: $(a+b) \neq \underbrace{\frac{\pi}{2} + \pi k}_{\text{The Asymptote Family}}, k \in I$

Proof: L.S. $= \tan(a+b)$
$= \frac{\sin(a+b)}{\cos(a+b)}$
$= \frac{\sin a \cos b + \sin b \cos a}{\cos a \cos b - \sin a \sin b}$
$= \frac{\frac{\sin a \cos b + \sin b \cos a}{\cos a \cos b}}{\frac{\cos a \cos b - \sin a \sin b}{\cos a \cos b}}$
$= \frac{\frac{\sin a \cancel{\cos b}}{\cos a \cancel{\cos b}} + \frac{\sin b \cancel{\cos a}}{\cancel{\cos a} \cos b}}{\frac{\cancel{\cos a \cos b}}{\cancel{\cos a \cos b}} - \frac{\sin a \sin b}{\cos a \cos b}}$
$= \frac{\tan a + \tan b}{1 - \tan a \tan b}$

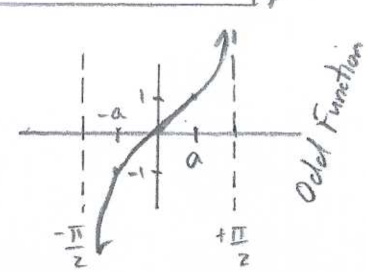
RS $= \frac{\tan a + \tan b}{1 - \tan a \tan b}$

$\because$ LS = RS
$\therefore$ Proven.

Remark$_2$: Difference Formula For Tangent: $\boxed{\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}}$

Technically: $(a-b) \neq \frac{\pi}{2} + \pi k, k \in I$ so our answers can exist finitely.

Proof: L.S. $= \tan(a-b) = \tan(a + (-b))$
$= \frac{\tan(a) + \tan(-b)}{1 - \tan(a)\tan(-b)}$
$= \frac{\tan a - \tan b}{1 + \tan a \tan b}$
$=$ R.S. ✓



$\because \tan(a) = -1\tan(-a)$
$\therefore -1\tan a = \tan(-a)$

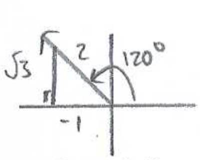
e)

Ex, Evaluate $\tan \frac{11\pi}{12}$ exactly

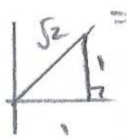
Sol'n,

$= \tan 165°$

$= \tan(120+45)$, added and subtracted special angles to obtain 165°.



RAA = 60°



$= \dfrac{\tan 120 + \tan 45}{1 - \tan 120 \tan 45}$

$= \dfrac{\left(\frac{\sqrt{3}}{-1}\right) + (1)}{1 - \left(\frac{\sqrt{3}}{-1}\right)(1)}$

$= \dfrac{-\sqrt{3}+1}{1+\sqrt{3}} \times \dfrac{(1-\sqrt{3})}{(1-\sqrt{3})}$, to rationalize denom

$= \dfrac{-\sqrt{3}+3+1-\sqrt{3}}{1-\sqrt{3}+\sqrt{3}-3}$, FOIL numer and denom

$= \dfrac{4-2\sqrt{3}}{-2}$

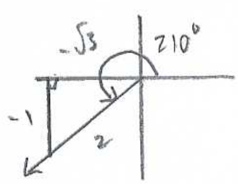
$= -2+\sqrt{3}$, tidy up division

Sol'n₂

$\tan \frac{11\pi}{12}$

$= \tan(210-45)$

$= \dfrac{\tan 210 - \tan 45}{1 + \tan 210 \tan 45}$



RAA = 30°

$= \dfrac{\left(\frac{-1}{-\sqrt{3}}\right) - (1)}{1 + \left(\frac{-1}{-\sqrt{3}}\right)(1)}$

$= \dfrac{\frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{3}}}{\left(\frac{\sqrt{3}}{\sqrt{3}}\right) \times 1 + \frac{1}{\sqrt{3}}}$

To build common denom

$= \dfrac{\frac{1-\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$

$= \dfrac{1-\sqrt{3}}{\cancel{\sqrt{3}}_1} \times \dfrac{\cancel{\sqrt{3}}^1}{\sqrt{3}+1}$, copy-flip-multiply

$= \dfrac{1-\sqrt{3}}{\sqrt{3}+1} \times \dfrac{(1-\sqrt{3})}{(1-\sqrt{3})}$, to rationalize denom

$= \dfrac{1-2\sqrt{3}+3}{\sqrt{3}-3+1-\sqrt{3}}$, FOIL numer and denom

$= \dfrac{4-2\sqrt{3}}{-2}$

$= -2+\sqrt{3}$, tidy up.

W.S. Compound Angles - Pg 2.
[Answers on Pg 2 bottom.]
Every Other Letter ✓