

A 8.4 * Solving Linear Trig Equations With Radians *

Warmup: 1. Evaluate exactly. (No calculators.)

a) $\cot 450^\circ$

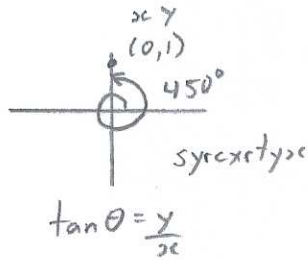
Sol'n

$$= \frac{1}{\tan 450^\circ}$$

$$= \frac{1}{\left(\frac{1}{0}\right)}$$

$$= 1 \times \frac{0}{1}$$

$$= 0$$



b) $\sin 540^\circ$

Sol'n

$$= \frac{y}{r}$$

$$= \frac{0}{1}$$

$$= 0$$

Think: Unit Circle
($r=1$)

Remark: There are several strategies to consider when solving trig equations:

- 1) Factor when this is possible.
- 2) Use the standard aka parent curves $y = \sin \theta$, $y = \cos \theta$, $y = \tan \theta$.
- 3) Locate $\angle \theta$ on a grid using the CAST rule.
- 4) Note the restrictions on the variable. That is, the RADIAN DOMAIN is important.
- 5) SyrCxrTyc and Unit Circle.

Ex₁ Solve $7 \cos x + 1 = 3(2 \cos x + \frac{2}{3})$, $0 \leq x \leq 2\pi$

Sol'n

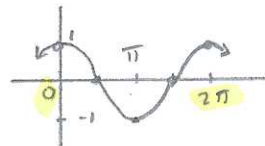
$$7 \cos x + 1 = 6 \cos x + 2$$

$$\therefore \cos x + 1 = 2$$

$$\cos x = 1$$

$$\therefore x = \cos^{-1}(1)$$

$$x = 0, 2\pi$$



Roots are 0 and 2π .

Ex₂ Solve $2 \sin \theta + 7 = 6 \sin \theta + 5$, $0 \leq \theta \leq 2\pi$

Sol'n

$$7 = 4 \sin \theta + 5$$

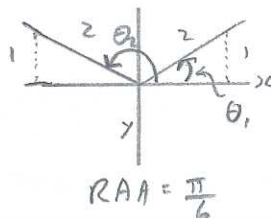
$$2 = 4 \sin \theta$$

$$\therefore \sin \theta = \frac{1}{2}$$

$$\theta_1 = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\hat{=} 0.52^r$$

$$= \frac{\pi}{6}$$



OR $\theta_2 = \pi - \frac{\pi}{6}$

$$= \frac{5\pi}{6}$$

$$\hat{=} 2.62^r$$

\therefore Solutions are about 0.52^r and 2.62^r ✓.

Ex₃ Solve a) $\csc \theta + 1 = 0$, $0 \leq \theta < 2\pi$.

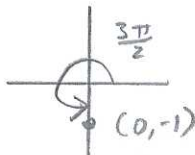
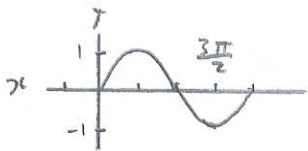
Sol'n

$$\csc \theta = -1$$

$$\therefore \frac{1}{\sin \theta} = -1, \text{ by def'n.}$$

$$\therefore \frac{\sin \theta}{1} = \frac{-1}{1}, \text{ flipped both sides}$$

$$\therefore \sin \theta = -1$$



$$\sin \theta = \frac{y}{r}$$

$$\therefore -1 = \frac{y}{1}$$

$$\therefore y = -1$$

$$\therefore \theta = \frac{3\pi}{2}$$

Think parent curve or unit circle

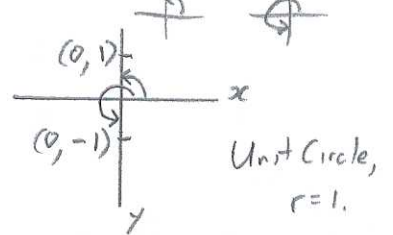
b) $\tan \theta = \infty$, $0 \leq \theta < 2\pi$

Sol'n $\therefore \tan \theta = \frac{y}{x}$

$$\therefore \frac{y}{x} = \infty$$

$$\therefore x = 0$$

$$\therefore \theta = \frac{\pi}{2} \text{ and } \frac{3\pi}{2}$$



Ex₄ Solve. $0 \leq x < 360^\circ$

a) $\cot x - 6 = 0$

Sol'n $\cot x = 6$

"flip-flip" $\rightarrow \tan x \approx 0.1667$

$$\therefore x = \tan^{-1}(0.1667)$$

$$x \approx 9.46^\circ$$

or

$$x = 180 + 9.46^\circ$$

$$\approx 189.46^\circ$$

$$\therefore \text{Solution Set} = \{9.46^\circ, 189.46^\circ\}$$

A P320* (10, 11) ab

W.S. # (1, 3) every other letter

#4 abd.

b) $2 \csc x - 3 = -15$

Sol'n $2 \csc x = -12$

$$\csc x = -6$$

$$\therefore \sin x = \frac{1}{-6}, \text{ "flip-flip"}$$

$$\therefore \sin x \approx -0.1667$$

$$\therefore x = \sin^{-1}(-0.1667)$$

$$x \approx -9.59^\circ$$

$$\text{So RAA} = 9.59^\circ$$

$$\text{So } x_1 = 180 + 9.59^\circ$$

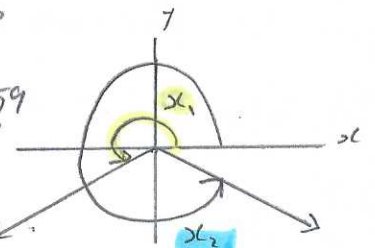
$$\approx 189.59^\circ$$

and

$$x_2 = 360 - 9.59^\circ$$

$$\approx 350.41^\circ$$

\therefore Root List is $\{189.59^\circ, 350.41^\circ\}$ to 2 d.p.'s.



Solving Linear Trigonometric Equations

- Solve each of the following equations to two decimal places, $0 \leq \theta \leq 2\pi$.

a) $\sin\theta = 0.3124$	b) $\cos\theta = 0.7315$	c) $\tan\theta = 3.1571$
d) $\sin\theta = -0.8135$	e) $\cos\theta = -0.1476$	f) $\tan\theta = -0.3541$
- For parts d, e, and f of question 1, rewrite your answers if $\theta \in \mathbb{R}$.
- Solve each of the following equations. Give exact answers only, $0 \leq \theta \leq 2\pi$.

a) $\sin\theta = \frac{\sqrt{3}}{2}$	b) $\cos\theta = -\frac{1}{\sqrt{2}}$	c) $\tan\theta = \sqrt{3}$
d) $\csc\theta = -2$	e) $\tan\theta = -1$	f) $\cos\theta = \frac{1}{2}$
- Solve each of the following equations. State the exact answers if possible. Otherwise, write your answers correct to two decimal places. Be careful with the domain of the solutions.
 - $5\tan x - 3 = 3\tan x + 7, 0 \leq x \leq 2\pi$.
 - $7(\cos x + 5) = 13 + 3(\cos x + 8), 0 \leq x \leq 2\pi$.
 - $6 \sin(x) - 4 = 2 \sin(x) + 1, -\pi \leq x \leq \pi$.
 - $5 + 4\cos x = 2, 0 \leq x \leq \pi$.
 - $4\sqrt{2} \tan x - 11 = \sqrt{2} \tan x - 9, \pi \leq x \leq 2\pi$
- Solve each of the following equations. State exact answers if possible. Otherwise, round your answers correctly to two decimal places.
 - $\sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}, 0 \leq x \leq 2\pi$.
 - $5 \tan\left(x - \frac{\pi}{3}\right) + 3 = 1, 0 \leq x \leq 2\pi$.
 - $2 \cos\left(x + \frac{5\pi}{6}\right) + 4 = 3, 0 \leq x \leq 2\pi$.
- Solve for $x, 0 \leq x \leq 2\pi$. Give exact answers.

a) $(2\sin x + 1)(\cos x - 1) = 0$	b) $4\cos^2 x - 1 = 0$
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Answers: 1. a) 0.32, 2.82 b) 0.75, 5.53 c) 1.26, 4.41 d) 4.09, 5.33

e) 1.72, 4.56 f) 2.80, 5.94 2. d) $4.09 + 2k\pi, 5.33 + 2k\pi, k \in \mathbb{I}$

e) $1.72 + 2k\pi, 4.56 + 2k\pi, k \in \mathbb{I}$ f) $2.80 + k\pi, 5.94 + k\pi, k \in \mathbb{I}$

3. a) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$ b) $\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$ c) $\left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}$ d) $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ e) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ f) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

4. a) 1.37, 4.51 b) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ c) no solution d) 2.42 e) 3.58

5. a) $\left\{\frac{\pi}{12}, \frac{5\pi}{12}\right\}$ b) 0.67, 3.81 c) $\left\{\frac{\pi}{2}, \frac{11\pi}{6}\right\}$ 6. a) $\left\{0, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi\right\}$ b) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$